Analytical Models for Strength and Stiffness Evaluation of Concrete Structures

Modèles pour l'évaluation de la résistance et la rigidité des structures en béton

Modelle für die Berechnung von Tragfähigkeit und Steifigkeit von Betontragwerken

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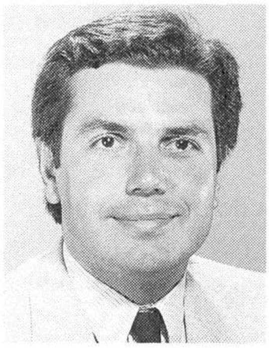

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SUMMARY

The development of rational measures for strengthening and retrofitting of existing concrete structures depends on advanced methods of assessing their strength and stiffness. These methods should be capable of predicting the future behavior of the entire structure based on information about the original design and the current state of the structure. This paper presents a general frame member model based on the fiber concept and capable of simulating the hysteretic behavior of concrete members under arbitrary histories of biaxial moment and axial force.

RÉSUMÉ

Les moyens de renforcement et de remise en état de structures en béton se basent sur des méthodes avancées pour établir la résistance et la rigidité de ces structures. Ces méthodes devraient être capables de prévoir le comportement de toute la structure à partir d'informations tirées du projet original et de l'état actuel de la structure. Le présent article propose un modèle général pour l'étude de cadres. Ce modèle repose sur le concept de fibre et peut reproduire le comportement histérétique d'éléments en béton sous l'action d'une flexion biaxiale et d'une force axiale arbitraires dans le temps.

ZUSAMMENFASSUNG

Die Entwicklung rationaler Methoden für die Verstärkung und Instandsetzung von gebauten Betontragwerken hängt von Modellen ab, die imstande sind, die Tragfähigkeit und Steifigkeit ihrer Tragelemente vorauszusagen. Diese Methoden sollten imstande sein, ausgehend von Information über den Anfangs- und jetzigen Zustand des Tragwerks, das zukünftige Verhalten vorauszusagen. Diese Arbeit stellt ein allgemeines Model für Rahmenelemente unter allgemeiner Belastungsgeschichte von zweiachsiger Biegung und Normalkraft vor.

1. INTRODUCTION

The development of rational measures for strengthening and retrofitting of existing concrete structures depends on advanced methods of assessing their strength and stiffness. These methods should be capable of predicting the future behavior of the entire structure based on information about the original design and the current state of the structure. The latter can be usually approximated from current measurements of material and structural properties using system identification methods. In regions of high seismic risk the difficulty of the problem is compounded by the complex loading history of existing structures, which might have experienced several small and moderate earthquake excitations in their service life.

The evaluation of the future behavior of existing concrete structures depends on the development of advanced analytical models, which describe the time and load dependent nonlinear behavior of the structural members. These models should satisfy two basic requirements: (a) they should be reliable, robust and computationally efficient and (b) they should be of variable complexity depending on the degree of detail required from the analysis: while individual critical members of the structure can be evaluated with sophisticated finite element models, the behavior of multistory buildings and multiple span freeway structures can be described with sufficient accuracy with member models. In fact, the ability to mix finite element models of critical regions of the structure with nonlinear or even linear member models of the rest of the structure should be an important consideration in the development of such models.

In the following, a new fiber beam-column finite element for the analysis of reinforced concrete structures is presented. Contrary to most existing beam finite elements which are based on the definition of displacement shape functions, the element described herein assumes a constant axial force and linear bending moment diagrams inside the element, thus assuming force shape functions. A general overview of the element formulation and of the element nonlinear iteration scheme needed for the element state determination is first presented, followed by the description of a few numerical examples in which the element response is compared with experimental results

2. ELEMENT FORMULATION

The beam-column element is shown in the local reference system x,y,z in Fig. 1. The element is represented without rigid-body modes, thus forces and deformations are measured with respect to the cord connecting the two end nodes. Forces and displacements are grouped in the following vectors:

$$\text{Element force vector} \qquad \mathbf{Q} = \{Q_1 \quad Q_2 \quad Q_3 \quad Q_4 \quad Q_5\}^T \tag{1}$$

$$\text{Element displacement vector} \qquad \mathbf{q} = \{q_1 \quad q_2 \quad q_3 \quad q_4 \quad q_5\}^T \tag{2}$$

Similarly, section forces and deformations can be grouped in the vectors:

$$\text{Section force vector} \qquad \mathbf{D}(x) = \{M_1(x) \quad M_2(x) \quad N(x)\}^T \tag{3}$$

$$\text{Section deformation vector} \qquad \mathbf{d}(x) = \{\chi_1(x) \quad \chi_2(x) \quad \bar{\varepsilon}(x)\}^T \tag{4}$$

The element is composed of a finite number of longitudinal fibers Each cross section is therefore described by the number of fibers, their area, location and force-deformation relations. Since the element has been developed for the analysis of reinforced concrete structures, concrete and steel constitutive models have been used [1]. Small kinematics are postulated and plane sections are assumed to remain plane and normal to the longitudinal axis. Consequently, the effects of shear and bond-slip are neglected in the present model. The nonlinear nature of the problem depends entirely on the nonlinear fiber force-deformation relations. The element formulation is based on the assumption that the axial force is constant and the bending moment diagram is linear inside the element. In symbols this translates to a simple relation between section and element forces:

$$\mathbf{D}(x)=\mathbf{b}(x)\cdot\mathbf{Q} \qquad \text{and} \qquad \Delta\mathbf{D}(x)=\mathbf{b}(x)\cdot\Delta\mathbf{Q} \tag{5}$$

where Δ denotes increments and $\mathbf{b}(x)$ is defined by:

$$\mathbf{b}(x)=\begin{bmatrix} \left(\frac{x}{L}-1\right) & \left(\frac{x}{L}\right) & 0 & 0 & 0 \\ 0 & 0 & \left(\frac{x}{L}-1\right) & \left(\frac{x}{L}\right) & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The force field, as defined by (5), is exact as long as only nodal forces act on the element. Loads acting inside the element can be easily introduced using the procedure described in [1].

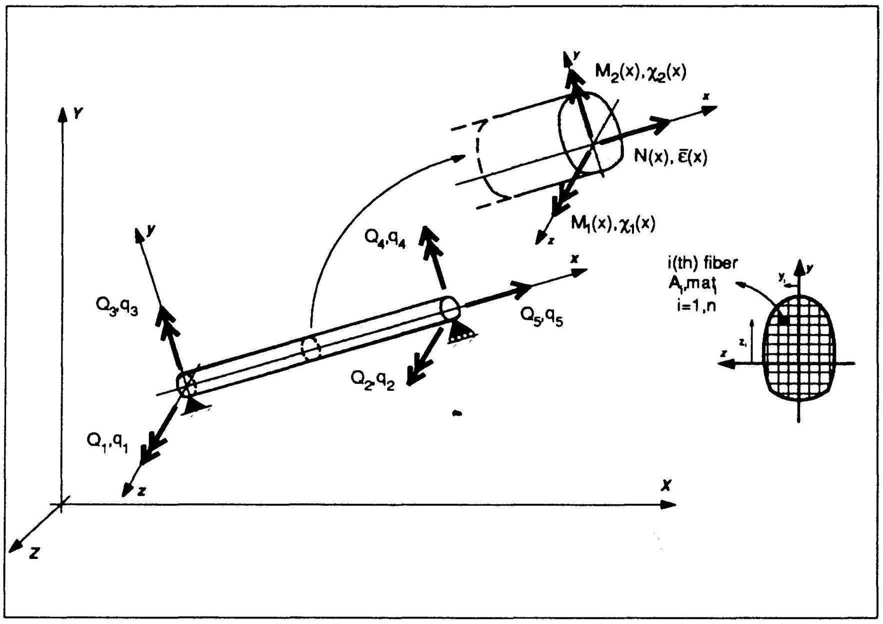

FIGURE 1 - BEAM ELEMENT FORCES AND DISPLACEMENTS WITHOUT RIGID BODY MODES IN LOCAL REFERENCE SYSTEM: FIBER DISCRETIZATION OF CROSS SECTIONS

The element is formulated using the flexibility method rather than the classical stiffness method, because of the advantage of defining an "exact" force field inside the element. Calling **P-Q** the element unbalanced forces (difference between applied and resisting forces **P** and **Q** respectively) and $\Delta\mathbf{q}$ the element deformation increments, the nonlinear system of equations at the element level is written:

$$[\mathbf{F}]^{-1}\cdot\Delta\mathbf{q}=(\mathbf{P}-\mathbf{Q}) \tag{7}$$

In Eq. 7 the element stiffness appears as the inverse of the element flexibility to indicate that the element is flexibility-based. The element flexibility matrix is determined integrating the section flexibilities according to:

$$\mathbf{F}=\left[\int_0^L \mathbf{b}^T(x)\cdot\mathbf{f}(x)\cdot\mathbf{b}(x)\cdot dx\right] \tag{8}$$

Section flexibility is obtained by inverting the section stiffness. The element is implemented in a stand alone program organized along the lines of a typical finite element code. Loads are applied on the structure and the program computes the corresponding displacements. The nonlinear solution procedure is organized as follows:

Load increments $\Delta \mathbf{P}$ are applied to the structure and a Newton-Raphson scheme is used to compute the corresponding structure displacement increments. At every Newton-Raphson iteration it is necessary to determine the element resisting forces corresponding to the updated element displacements. This is a challenging task when working with a flexibility-based element, because force and not displacement shape functions must be used. A new scheme has been developed for the proposed element, based on residual section and element deformations. Given the updated element displacements, the following steps are performed:

1) Compute the element linearized force increments using the last computed element tangent stiffness, and update the element forces;
2) Compute the section force increment using (5);
3) Compute the section deformation increment using the last computed section flexibility;
4) From the new section deformations, using the hypothesis that plane sections remain plane and normal to the longitudinal axis, compute the new fiber strains;
5) Compute fiber stresses and tangent moduli using the fiber force-deformation relations;
6) Compute the new section tangent stiffness, the section resisting forces and the section unbalanced forces, difference between applied and resisting forces;
7) Transform the section unbalanced forces into section residual deformations using the section flexibility;
8) Integrate the section residual deformations to compute the element residual deformations;
9) Compute the element flexibility using (8);
10) Compute the new element force increments;

Step 10) is needed because the element residual deformations can not be applied to the element alone, otherwise node compatibility would be violated. Forces based on the new element stiffness are applied to the element in order to yield element displacements equal and opposite to the element residual deformations. Correspondingly, force and deformation increments are applied to all sections: these increments are computed repeating steps 3) through 9) until convergence is achieved. The element converges when the unbalanced forces at all sections are sufficiently small. Element convergence implies that the element resisting forces corresponding to the applied displacements have been computed and the following Newton-Raphson iteration can be performed.

The new element convergence scheme is based on the equilibrium conditions (5). It can be shown that during the iterations equilibrium and convergence inside the element is respected, and section force-deformation relations are satisfied, at least within a certain tolerance, when convergence is reached. More details on the approach and a thorough description of the iteration scheme are presented in [1].

3. EXAMPLES

A series of comparisons between analytical and experimental results are used to study the element performance. Four examples are illustrated in this section: these refer to three reinforced concrete cantilevers discretized with a single beam-column element. Displacement control techniques have been used to match experimental and analytically imposed displacements: a very strong linear elastic spring has been positioned at free end of the cantilever to control the tip displacements.

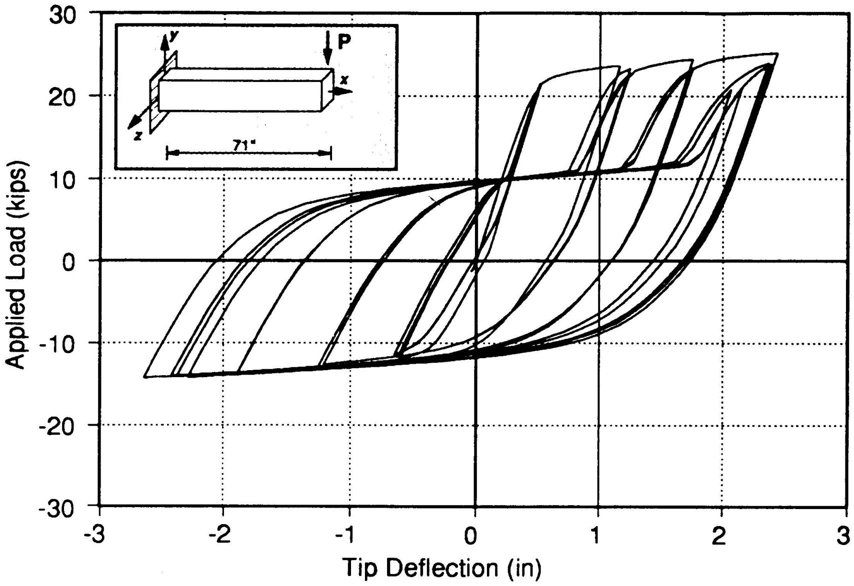

FIGURE 2 - TIP LOAD-DISPLACEMENT RESPONSE OF CANTILEVER BEAM UNDER CYCLIC UNIAXIAL BENDING

The first example shows the uniaxial bending of a cantilever beam R1 with a rectangular cross section tested in [2]. The simulation of the tip displacement response in the strong direction *y* is shown in Fig. 2. Analytical and experimental results agree well, especially for displacements up to yielding of the built-in end. At this point bond-slip and shear deformations become important and since the element does not include such effects, the analytical and experimental results show some discrepancy.

The remaining examples refer to the bending behavior of a cantilever under a compressive axial load and biaxial or uniaxial bending moments, which was tested in [3]. Fig. 3 illustrates the uniaxial case in which a constant axial force and a cyclic force along the weak axis *z* are applied at the tip of the cantilever. Displacement control was not used in this example. Numerical and experimental results are very similar and show a stiffer fiber model behavior, especially for low levels of lateral force at which "pinching" is evident in the experimental results.

The same cantilever is studied under biaxial bending conditions. Two cyclic transverse loads are applied at the free end of the cantilever. Displacement control is used in this example. Fig. 4 shows the tip response in the strong direction *y*. The correlation between analytical and experimental results is very good both for small and large displacements. When the concrete is fully cracked at the built-in section, bond-slip effects appear in the experimental data, but their contribution to the tip displacements is small.

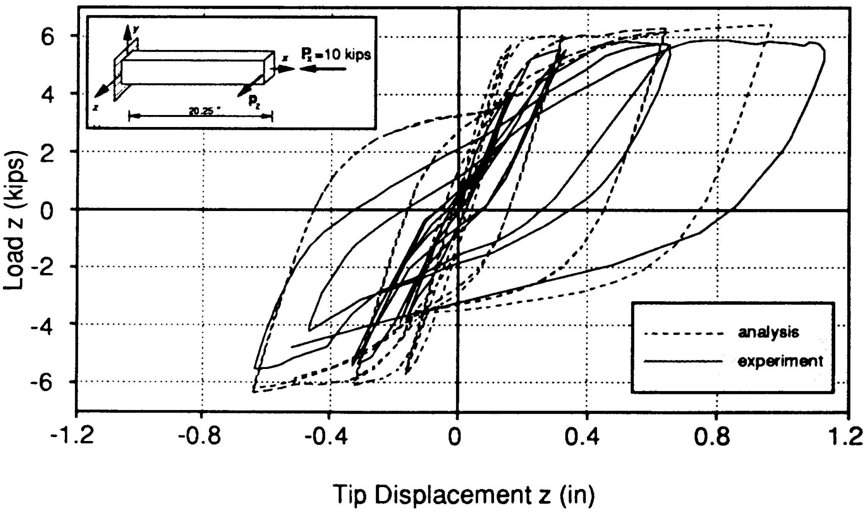

FIGURE 3 - TIP LOAD-DISPLACEMENT RESPONSE OF CANTILEVER UNDER CONSTANT AXIAL LOAD AND CYCLIC UNIAXIAL BENDING: NUMERICAL AND EXPERIMENTAL RESULTS

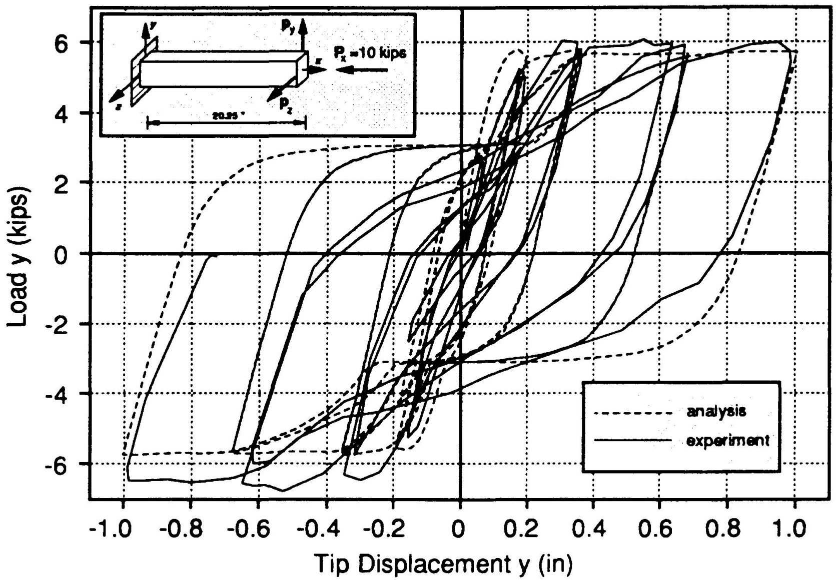

FIGURE 4 - TIP LOAD-DISPLACEMENT RESPONSE IN THE STRONG DIRECTION y OF A CANTILEVER BEAM UNDER CONSTANT AXIAL LOAD AND CYCLIC BIAXIAL BENDING: NUMERICAL AND EXPERIMENTAL RESULTS.

Finally, the same cantilever beam is studied under both cyclic bending and cyclic axial force. According to the notation of Fig. 5, the following load and displacement histories have been imposed:

$$\mathbf{P}_x = -75 \pm 30 kips$$
$$\mathbf{p}_y = \pm 0.96 in$$
$$\mathbf{p}_z = \pm 0.96 in$$

Load and deformation increments are applied so that cycles are simultaneous: all three quantities reach their maximum and minimum values at the same time. This example is particularly important to show the capability of the proposed element to represent softening and stiffness degradation without any computational difficulties. This is due to the fact that force equilibrium is always maintained along the element. When softening initiates at the built-in section, the whole beam unloads respecting the prescribed linear bending moment diagrams. All sections unload elastically except for the built-in section, which softens. Correspondingly, the end curvature increases while curvatures at all other sections decrease.

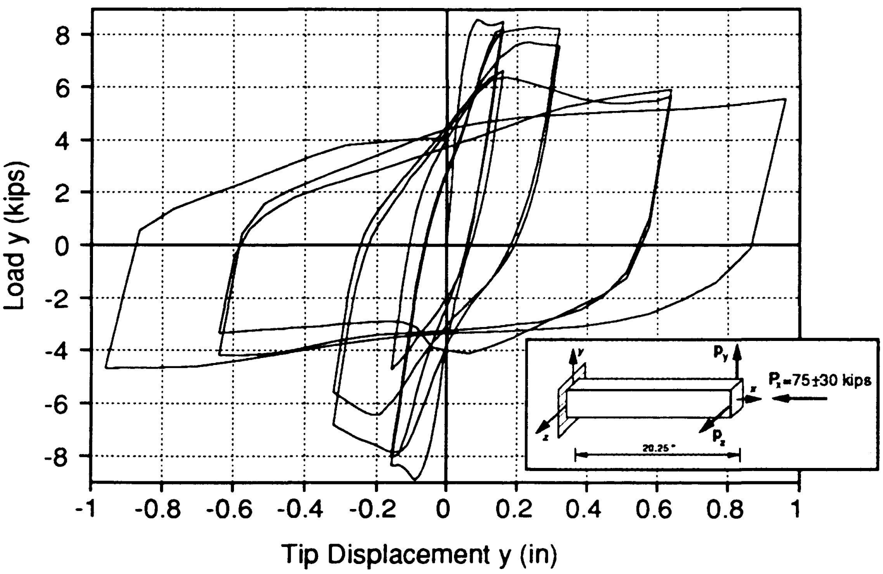

FIGURE 5 - TIP LOAD-DISPLACEMENT RESPONSE IN STRONG DIRECTION y OF CANTILEVER UNDER CYCLIC AXIAL LOAD AND CYCLIC BIAXIAL BENDING

4. CONCLUSIONS

To predict the response of existing reinforced concrete structures to strong ground motions and to develop better strengthening and retrofit measures for structures in zones of high seismic risk integrated experimental and analytical studies are very important. The beam-column fiber element presented in this paper is part of an ongoing effort to develop reliable computational tools of different levels of complexity and, thus, reliability, for modeling reinforced concrete structures. Most two-node reinforced concrete finite elements are based on the stiffness approach which postulates linear curvatures and constant axial strain along the element. These deformation distributions do not represent the physical behavior when sections start yielding. The proposed finite element is based on the assumption of linear bending moment diagram and constant axial force along the element. This hypothesis is exact when no load is applied inside the element. The computational cost for each element is higher when a flexibility based element is used, because of the iteration scheme necessary to compute the element resisting forces corresponding to the applied displacements. However, fewer elements are needed to discretize the structure, thus requiring a smaller number of total degrees of freedom. Further refinements of the element are needed to include bond-slip and second order effects.

5. REFERENCES

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