# Reliability analysis of an existing bridge

# Reliability Analysis of an Existing Bridge
## Analyse de la fiabilité d'un pont existant
## Zuverlässigkeitsanalyse einer existierenden Brücke

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**SUMMARY**

The assessment is described of the remaining structural capacity of an existing concrete bridge. A probabilistic reliability analysis is applied to a simple conventional carrying capacity model for the bridge. This simplified reliability analysis is calibrated by a random effectivity factor to give realistic results. The calibration uses some particularly chosen deterministic analyses of the bridge. These analyses are based on a refined FEM-model of the failure behaviour taking into account that the observed strength throughout the structure differs from what was assumed at the design stage. The cases for deterministic analysis are obtained through the reliability analyses of the simple model.

**RÉSUMÉ**

L'article traite de l'évaluation de la résistance restante d'un pont en béton armé. L'analyse probabiliste de la fiabilité du pont est réalisé sur la base d'un modèle simple de la résistance ultime du pont. Cette analyse simplifiée de la fiabilité est calibrée au moyen d'un facteur d'efficacité pour obtenir des résultats exacts et réalistes. Le calibrage utilise des résultats de certaines analyses déterministes des structures du pont. Ces analyses ont été faites en utilisant un modèle très détaillé, par éléments finis du comportement du pont en tenant compte que la résistance observée en certaines parties du pont est différente de celles supposées lors de l'établissement du projet. L'analyse déterministe est établie sur la base de l'analyse de la fiabilité du modèle simplifié.

**ZUSAMMENFASSUNG**

Die Bewertung der Resttragfähigkeit einer Betonbrücke wird beschrieben. Eine Wahrscheinlichkeitsanalyse der Sicherheit ist auf der Grundlage eines einfachen Standardmodells der Tragfähigkeit der Brücke durchgeführt. Diese einfache Wahrscheinlichkeitsanalyse ist mit einem Effektivitätsfaktor kalibriert, um ein realistiches Ergebnis zu erreichen. Die Kalibrierung nutzt speziell ausgewählte deterministiche Analysen der Brücke. Diese basieren auf einem verfeinerten FEM-Modell bezüglich der Brückenkonstruktion unter Berücksichtigung von Festigkeitsabweichungen gegenüber den Bemessungsannahmen. Die betreffenden deterministischen Untersuchungen werden aufgrund des einfachen Zuverlässigkeitsmodells bestimmt.

## 1. Introduction

Well–developed rational reliability based methods for designing new concrete bridges are available today. However, for a number of reasons this is not the case concerning the assessment of the remaining structural capacity of an existing and deteriorated bridge.

A reliability analysis of a bridge in the design state is a formal procedure based on common practice. The models have to a certain extent become standard so that the target safety levels together with associated and selected failure modes give structural dimensions which are known to be satisfactory for normal structures. Moreover, the analytical models used in design are practically manageable in size and complexity and they are assumed to model the structural carrying properties of the bridge in a sufficiently realistic way.

A similar standard procedure for analysing existing bridge structures has not yet been developed. When considering an existing bridge the reliability analysis is no longer just a formal procedure. The potential failure modes have to be modelled realistically taking into account the available knowledge on geometry, strengths, etc. This raises the problem of how to set up such probabilistic models that are sufficiently rich in concepts to take the available information into account and at the same time can be standardized to an extent that makes the reliability analysis result comparable with the result from a similar reliability analysis of another existing bridge or with specified target safety levels.

Another problem is that the reliability analysis of an existing bridge due to observed deteriorations, errors etc. often becomes more complicated than the analysis during the design state. However, such information must be considered seriously and it makes it more difficult to set up models that are practically manageable.

In the following a method is demonstrated by which these problems can be overcome for concrete bridges suffering from severe damages.

## 2. The considered existing bridge

The bridge across Salpetermosevej in Hillerød, Denmark, was constructed in 1977. It is designed as a reinforced concrete frame structure. The length of the free span is approximately 6 m.

The concrete used for casting the bridge was supplied by a local plant for ready–mixed concrete and delivered by truck mixers. The workability of the concrete was very poor and too stiff for the contractor to obtain a satisfactory compaction. Thus, the hardened concrete obtained gross porosity, showing high intensity of honeycomb at the finished concrete surface and a high content of entrapped air in the interior of the structure. Furthermore, the fresh concrete even contained fractions of hardened concrete due to insufficient cleaning of the mixer. The compressive strength of the concrete was determined by cast cylinders. The test results indicated that the potential compressive strength of the concrete in the structure would be lower than prescribed, mainly due to large variability. An investigation made in 1977 with tests on drilled cores from the bridge verified this suspicion.

The present appearance of the bridge shows concrete which is seriously disintegrated by cracks and other signs of deterioration especially in the bridge deck. Due to the extensive porosity of the concrete the influence of aggressive substances from the environment is significant. The carbonation, the chloride ingress (de–icing salt) and the leaching by rainwater seeping through the concrete have been the dominating environmental actions on the concrete bridge. The effect of these attacks is a decrease of the compressive strength and a loss of protection against corrosion of the rebars in various parts of the structure. In this paper we will be content with a study of the reliability analysis of the deteriorated bridge deck.

## 3. The effectivity factor reliability analysis method

The probabilistic reliability analysis of the bridge deck can be made on the basis of a simple conventional upper bound yield line collapse model. In the following we will denote this model as the simple model. In order to make the results of the analysis "realistic" an effectivity factor reduction is introduced on some of the variables from which the yield moments are calculated. The effectivity factor is calculated from one or more carrying capacity results that correspond to certain optimized statically admissible stress fields. These statically admissible stress fields are obtained in a finite element model that represents a refined model of the local strength properties of the bridge deck taking into account that the yield moments vary over the deck according to some stochastic field model. This stochastic field model reflects the observed deteriorations of the bridge deck. Moreover, by using the lower bound theorem of the ideal plasticity theory searching an optimal admissible stress field it is automatically ensured that the reliability is obtained for the most critical failure mode. This finite element model will in the following be denoted as the elaborate model.

The inverted commas around the word realistic are put there because the ideal plasticity theory is not necessarily particularly realistic. However, the ideal plasticity theory is used herein in order to illustrate that a simple model by use of an effectivity factor modification can be made reliability equivalent to a far more elaborate model of the same phenomenon. The effectivity factor is obtained by a single or some few calculations with the elaborate model. The set of input values for these calculations with the elaborate model is obtained by a reliability analysis carried out by use of the simple model.

In this way an elaborate (and possibly realistic) model for the carrying properties of a structure can be reliability analysed by use of a suitably calibrated simple conventional carrying capacity model. This reliability equivalence may be the key to a rational codification of methods to evaluate remaining structural capacity. The theoretical considerations leading to the method are given in Ditlevsen and Arnbjerg–Nielsen (1992,1992). Here the method will be summarized without the argumentations for the validity of the method.

Let $(x_S, x_R, x_D)$ be the total vector of basic variables (input variables) that are contained in the elaborate model. The subvectors $x_S$ and $x_R$ are the vectors of load variables and strength variables respectively. These variables are with sufficient generality defined such that they all have physical units that are proportional to the unit of force. The subvectors $x_D$ is the vector of all the remaining basic variables (of type as geometrical and dimensionless basic variables).

Two limit state equations

$$g_r(x_S,x_R,x_D) = 0 \quad , \quad g_i(x_S,x_R,x_D) = 0 \tag{1}$$

are given representing the "realistic" model and the idealized model respectively. It is assumed that for each fixed $(x_S,x_R,x_D)$ the equations

$$g_r(\kappa_r x_S,x_R,x_D) = 0 \quad , \quad g_i(\kappa_i x_S,x_R,x_D) = 0 \tag{2}$$

can be solved uniquely with respect to $\kappa_r$ and $\kappa_i$ respectively. The solutions are $\kappa_r(x_S,x_R,x_D)$ and $\kappa_i(x_S,x_R,x_D)$. By using the physical property of dimension homogeneity it can be shown (Ditlevsen and Arnbjerg–Nielsen 1993) that the two equations

$$g_r(x_S,x_R,x_D) = 0 \quad , \quad g_i\left(x_S,\frac{\kappa_r(x_S,x_R,x_D)}{\kappa_i(x_S,x_R,x_D)}x_R,x_D\right) = 0 \tag{3}$$

are equivalent in the sense that the two set of points they define are identical. The idea of the effectivity factor method is to use a suitable simple approximation to the last equation in (3) in the reliability analysis in place of the first equation in (3). The point is to approximate the effectivity factor function $\nu(x_S,x_R,x_D) = \kappa_r(x_S,x_R,x_D)/\kappa_i(x_S,x_R,x_D)$ by a constant or at most an inhomogeneous linear function of $(x_S,x_R,x_D)$. The approximation is made such that it is particularly good within the region of the space that contributes the most to the failure probability. Let $(x_S^*,x_R^*,x_D^*)$ be a point of this region and let $\nu^* = \nu(x_S^*,x_R^*,x_D^*)$ . The equation

$$g_i(x_S,\nu^* x_R,x_D) = 0 \tag{4}$$

then defines an approximating limit state in the important region. The problem is now reduced to the problem of how to choose the point of approximation $(x_S^*,x_R^*,x_D^*)$ . The answer to this problem is given in the reliability theory. With a judgmentally chosen value $\nu_0$ of $\nu^*$ a first or second order reliability analysis (FORM or SORM, see e.g. Madsen, Krenk, and Lind (1986) or Ditlevsen and Madsen (1991)) is made with (4) as limit state. This analysis determines the most central point (the design point) $(x_{S1},x_{R1},x_{D1})$ and an approximate failure probability $p_1$ . Using that $\kappa_i(x_{S1},x_{R1},x_{D1}) = 1/\nu_0$ an improved value $\nu_1 = \nu_0\kappa_{r1}$ of $\nu^*$ is calculated where $\kappa_{r1} = \kappa_r(x_{S1},x_{R1},x_{D1})$. Then a new FORM or SORM analysis is made with (4) as limit state. This gives the most central point $(x_{S2},x_{R2},x_{D2})$ and the approximate failure probability $p_2$ . Proceeding iteratively in this way we get a sequence $(\kappa_{r1},p_1),(\kappa_{r2},p_2),...$ that may or may not be convergent. If the sequence is convergent in the first component it is also convergent in the second component and we have $\kappa_{r1},\kappa_{r2},.. \to 1$, $p_1,p_2,.. \to p$ where $p$ will be denoted as the *zero order approximation* to the probability of the failure event of the elaborate model.

If the sequence is not convergent we still can define the zero order approximation by simple interpolation to the value $\kappa_r = 1$ among points $(\kappa_r,\beta)$ ($\beta=-\Phi^{-1}(p)$, $\Phi$ = standardized normal distribution function) corresponding to the sequence or simply obtained for a series of different values of $\nu_0$.

A check of the goodness of the zero order approximation is made by replacing the effectivity factor function $\nu(x_S,x_R,x_D)$ by its first order Taylor expansion

$$\nu(x_S,x_R,x_D) = \tilde{\nu}(x_S,x_R,x_D) \simeq \nu^* + a'(x_S-x_S^*) + b'(x_R-x_R^*) + c'(x_D-x_D^*) \tag{5}$$

at the most central point $(x_S^*,x_R^*,x_D^*)$ corresponding to the limit state (4) with $\nu^*$ being the effectivity factor value corresponding to $\kappa_r = 1$. The numerical determination of the coefficients **a, b, c** requires that the values of $\nu(x_S,x_R,x_D)$ are known at least at as many points in the vicinity of $(x_S^*,x_R^*,x_D^*)$ as the number of variables in $(x_S,x_R,x_D)$. These values of $\nu$ are obtained by solving the equations (2) with respect to $\kappa_r$ and $\kappa_i$ respectively at each chosen point $(x_S,x_R,x_D)$.

With (5) substituted for $\kappa_r/\kappa_i$ into the last equation in (3) we get a limit state for which both the probability of failure and the value of $\kappa_r$ in general will be different from the probability $p$ and the value $\kappa_r = 1$ as obtained by the zero order approximation. However, by a unique scaling factor $k_r$ on the load vector $x_S$ we can achieve that the limit state defined by $g_i(k_r x_S, \nu(k_r x_S,x_R,x_D)x_R,x_D) = 0$ corresponds to the failure probability $p$. With the Taylor expansion (5) substituted into this equation we get the limit state equation $g_i(k_r x_S, \tilde{\nu}(k_r x_S,x_R,x_D)x_R,x_D) = 0$ for which we can determine $k_r$ by iterative application of FORM or SORM analysis such that the corresponding failure probability becomes $p$. The pair $(k_r,p)$ will be called the *first order approximation.* The size of the deviation of $k_r$ from 1 can then be used to judge the accuracy of the zero order approximation. Also the change of the most central point contributes to this judgment.

## 4. Reliability analysis of deteriorated concrete bridge

The slab structure of the Salpetermosevej bridge is shown in Figure 1. The slab is one span and clamped in both ends. Actually the slab is skew with skew reinforcement, but the skewness is relatively small – the angle between a free edge and a clamped edge is 82°. Finite element calculations verify that assuming orthogonal reinforcement and a rectangular slab shape gives a small relative error on the load carrying capacity. Only one load case with fixed load is considered in the present study. According to the rules for loads on Danish road bridges, Vejdirektoratet (Danish Road Directorate) (1984), the critical truck load on the undamaged slab structure is found to consist of two trucks as shown in Figure 1. A uniformly distributed load is also prescribed but is found to be negligible in this case. For reference later (when reliability index versus load parameter curves are found), it is mentioned here that the load parameter value corresponds to the prescribed characteristic load including corrections for dynamic loading. In order to reduce the computational efforts in this illustration the analysis is made solely on one half part of the slab structure utilizing the geometrical and loading symmetry, Figure 1 . This is made possible by prescribing the torsional moment to be zero along the symmetry line in the finite element model. (From a stochastic modelling point of view this symmetrization is not necessarily correct).

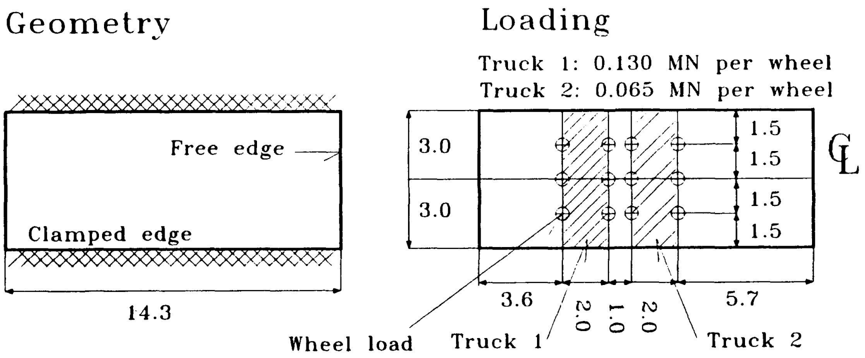



*Figure 1. The slab structure of the Salpetermosevej bridge. Length units =* [m].

In the elaborate model a lower bound solution is used. The analysis method is based on the lower bound theorem, which states that stress fields in equilibrium not violating the yield criterion are admissible solutions. The solution method is to find the stress distribution that maximizes the load obtained by proportional loading. Polygonization of the yield condition leads to a linear programming problem. A stress based finite element code is used, Høyer (1989). The FE code is described shortly in the following. The stress state is given by a set of stress parameters that always satisfy the local equilibrium conditions in an element, here a triangular 3–noded element. Global equilibrium is obtained in the system of nodal forces that correspond to the stress parameters and which are in equilibrium with the external nodal forces. The polygonized yield condition is checked at each node. The polygonized yield condition is given as

$$m_{xy} = \pm \min\{m_{F_x} - m_x, -m'_{F_x} + m_x, m_{F_y} - m_y, -m'_{F_y} + m_y\} \tag{6}$$

in which $m_x \in [-m'_{F_x}, m_{F_x}]$ is the moment per length unit in a cross section perpendicular to the x–axis and corresponding to compression in the upper side, $m_y \in [-m'_{F_y}, m_{F_y}]$ is defined analogously, while $m_{xy}$ is the torsional moment per length unit. Lower index F indicates yield capacity (absolute value) and prime indicates compression in the lower side of the slab.

It is noted that the polygonized yield surface (6) is inside the yield surface defined by

$m^2 = \min\{(m_{F_x}-m_x)(m_{F_y}-m_y),\ (m'_{F_x}+m_x)(m'_{F_y}+m_y)\}$ the latter being the standard yield surface for reinforced concrete slabs. Thus the polygonized yield surface leads to a lower bound solution as compared to the usual solution.

The simple model is based on the upper bound theorem in the theory of plasticity for ideal plastic materials. The work equation method is used, e.g. Nielsen (1984). A simple expression for the load carrying capacity of the undamaged homogeneous slab structure is set up as follows. The yield line pattern is shown in Figure 2. The fixed length of the positive yield line in the middle of the slab and the assumptions $d/x < 1/2$ and $x < d+b_T$ are found to be reasonable for the strength values of the undamaged slab structure. The load parameter $\lambda$ is then given by (for symbols see Figure 2) $\lambda P = (\alpha x^2+\beta x+\gamma)/(10x-d)$ where $\alpha = 8(m_{Fy}+m'_{Fy})/b$ , $\beta = 4b_T(m_{Fy}+m'_{Fy})/b$ , $\gamma = 2b(m_{Fx}+m'_{Fx})$ . The optimal value of $x$ is the relevant solution to the equation $11\alpha x^2-4\alpha dx-11\gamma-2\alpha d = 0$ .

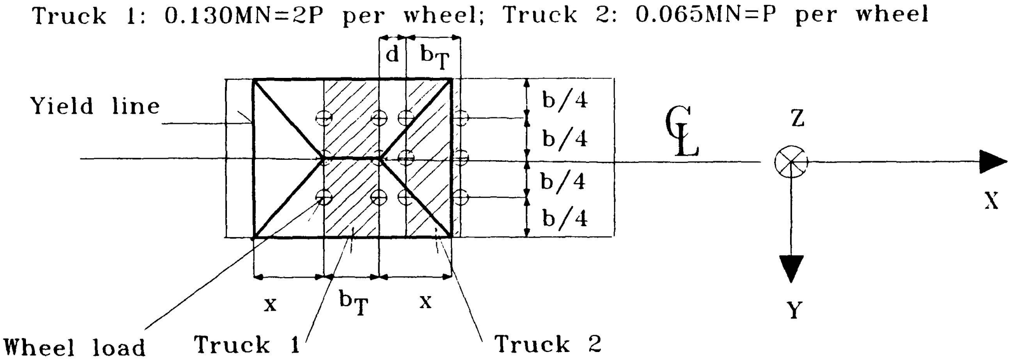



*Figure 2. Yield line pattern in upper bound calculation.*

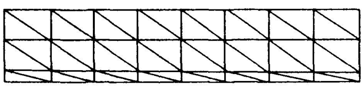
For the damaged structure the slab is nonhomogeneous. However, the same yield line pattern is used for optimization of the load parameter with respect to $x$ . The internal work is calculated approximately corresponding to the moment capacities in the different zones that model the damages of the slab structure.

The concrete strength is assumed constant over the thickness of the slab. Concrete covers and reinforcement areas are considered deterministic before the occurrence of damages. The yielding force of the reinforcement is used directly in the reliability analysis. The variables are: $f_c$ = concrete strength, $F_x$, $F_y$ = yield force per length unit in lower reinforcement in the x–direction and the y–direction respectively, $F'_x$, $F'_y$ = yield force per length unit in upper reinforcement in the x–direction and y–direction respectively, $d_x$, $d_y$ = effective depth of lower reinforcement in the x–direction and the y–direction respectively, $d'_x$, $d'_y$ = effective depth of upper reinforcement in the x–direction and the y–direction respectively.

As it is stated earlier, the finite element code is formulated in cross sectional moment capacities (per length unit). The bending moment capacities are calculated as for a normally reinforced beam, i.e. it is assumed that the reinforcement in tension is yielding at failure. The assumption is reasonable in a deterministic analysis considering fixed values only, for example characteristic values. In a reliability analysis one or more of the input variables can take such values in the tails of their respective distributions that other than the assumed modes of bending failure can occur. This matter is not persued further in this study. Neglecting reinforcement in compression, the moment capacities $m_{F_x}, m'_{F_x}, m_{F_y}, m'_{F_y}$ are given by a formula of the form $m_F = (1-\Phi/2)\Phi d^2 f_c$, $\Phi = F/(df_c)$ with the relevant indices x or y and no prime or prime put on all the symbols $m_F$, $\Phi$, d, F .

In the treated problem, the expressions (2) become $-\kappa_r x_S + \lambda_r(x_R, x_D) = 0$, $-\kappa_i x_S + \lambda_i(x_R, x_D) = 0$ where $\lambda_r(\cdot)$ and $\lambda_i(\cdot)$ are the carrying capacity functions corresponding to the elaborate and simple model, respectively. Hence the effectivity factor function simplifies to $\nu(x_S, x_R, x_D) = \lambda_r(x_R, x_D)/\lambda_i(x_R, x_D)$ showing that the effectivity factor is independent of the load. This gives the simplification relative to the general problem that solving the equation in (2) with respect to $\kappa_r$ and $\kappa_i$ requires only one calculation of $\lambda_r(x_R, x_D)$ and $\lambda_i(x_R, x_D)$ , respectively. Furthermore, derivatives with respect to the load variable need not to be calculated.

**Example 1: Corroded reinforcement. Minor cracking of the concrete** Corroded reinforcement and minor cracking of the concrete can be caused by chloride ingress. If only minor cracking with no sign of corrosion at the surface of the concrete has occurred, the corrosion is normally either limited or it has the character of pitting. Pitting can lead to a total loss of strength in a section. It is assumed in this example that severe corrosion damages are observed in a relatively large zone.



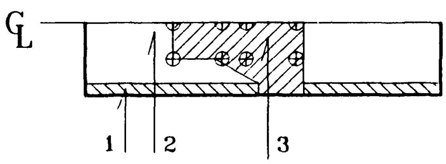



*Figure 3. Finite element mesh and zones corresponding to damages (half of the structure).*

The damage zones are shown in Figure 3. The damage zones are chosen to be the same as the damage zones for the actual slab of the Salpetermosvej bridge, treated in the next example, except that a much larger reduction of the lower reinforcement is assumed in zone 3. Zone 1 along the clamped edge is considered to be undamaged. The concrete strengths in the zones 2 and 3 are reduced by multiplying $f_c$ by the random variables $R_{fc2}$ and $R_{fc3}$ respectively. Analogously the reinforcement areas and thus the yielding forces in the lower side in zone 3 are reduced by the factors $R_{Fx3}$ and $R_{Fy3}$ .

The variables entering the problem and distribution assumptions are shown in Table 1. The units correspond to [m] and [MN]. Other geometrical properties of the slab are taken to be constant.

| Name | Distribution | Fixed value | Mean | C.o.v. |
|---|---|---|---|---|
| Load parameter | Fixed | Varying | - | |
| $f_c$ | Lognormal | - | 30.0 | 0.15 |
| $F_x, F'_x$ | Lognormal | - | 0.3848 | 0.05 |
| $F_y, F'_y$ | Lognormal | - | 0.8747 | 0.05 |
| $d_x, d'_x$ | Fixed | 0.245 | - | - |
| $d_y, d'_y$ | Fixed | 0.261 | - | - |
| $R_{fc2}$ | Lognormal | | 0.9 | 0.20 |
| $R_{fc3}$ | Lognormal | | 0.8 | 0.20 |
| $R_{Fx3}$ | Uniform:[0.4,0.6] | | - | - |
| $R_{Fy3}$ | Uniform:[0.3,0.5] | | - | - |
| all random variables are assumed to be mutually independent | | | | |

**Table 1** *Data for the reliability analysis. (C.o.v. = coefficient of variation)*

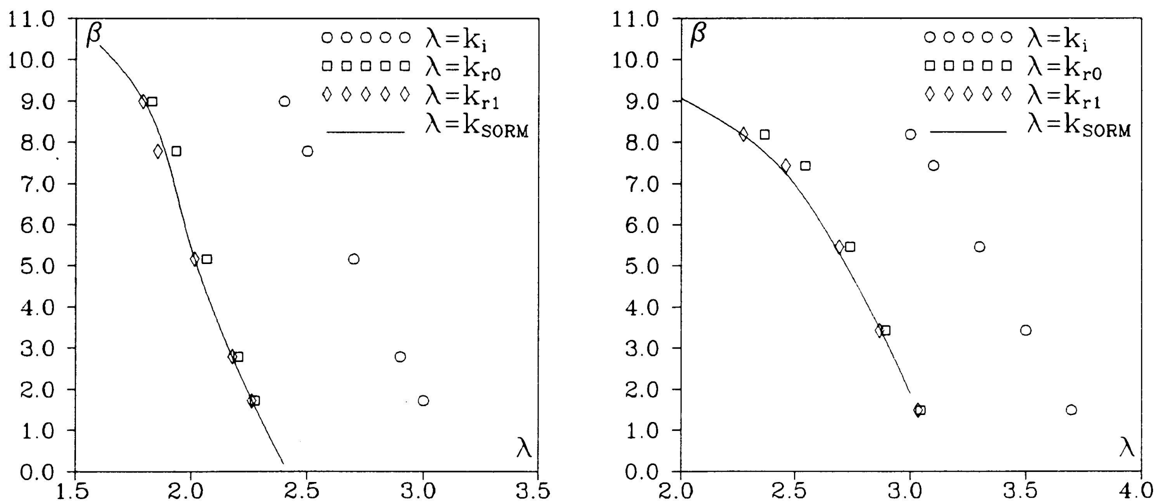



*Figure 4. Reliability index versus load parameter in Example 1 (left) and in the Example 2: Salpetermosevej (right).*

Corrections of the upper bound solution for a homogeneous slab are obtained by replacing $\alpha$, $\beta$, and $\gamma$ by (1, 2 and 3 refer to the zones as defined in Figure 3) $\alpha = 4(2m'_{Fx1}+m_{Fy2}+m_{Fy3})/b$ , $\beta = 4b_T(m'_{Fy}+m_{Fy3})/b$ , $\gamma = b(m_{Fx2}+m'_{Fx2}+m_{Fx3}+m'_{Fx3})$ . The results from the reliability analysis are shown in Figure 4 (left). The load parameter $k_i = 1/\nu^*$ corresponds to the simple model, whereas the load parameters $k_{r0}$ and $k_{r1}$ correspond to the effectivity factor method calculation with a constant and a first order Taylor expansion of the effectivity factor, respectively. The fully drawn curve comes from a direct SORM analysis for the elaborate model, i.e. the finite element model. It is seen that even with the large deviations between the results of the simple model and the elaborate model the effectivity factor method yields a quite good agreement between the zero order approximation results and the results from the elaborate model. Furthermore it is seen that the agreement is improved by using the first order approximation.

**Example 2 Salpetermosevej** Example 1 corresponds to the slab structure of the Salpetermosevej bridge with deliberately overestimated reinforcement reductions. In this example the data are the same as in Example 1 except for the random variables $R_{Fx3}$ and $R_{Fy3}$, i.e. the reduction factors of the yield forces in the lower reinforcement in the zone 3 in the directions $x$ and $y$ respectively. Here these reduction factors are assumed to be uniformly distributed between 0.9 and 1.0, that is, less severe reductions are assumed. Measurements of the present properties of the bridge have not been carried out but the bridge has been inspected visually. Based on engineering judgments it is anticipated that the assumed data very well can be valid for the bridge. The results from the reliability analysis are shown in Figure 4 (right). As in Example 1 there is good agreement between the effectivity factor method and results from the direct SORM analysis of the elaborate model.

## Acknowledgement

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