Probabilistic design concept

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Concepts probabilistes de calcul

Probabilistische Konzepte der Berechnung

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SUMMARY

Two kinds of uncertainties are to be generally distinguished when analyzing structural serviceability: randomness of basic variables and vagueness in definition of limit states. The second kind of uncertainty may be handled by methods of fuzzy set theory. Derived unserviceability measures enable one to formulate probabilistic design concepts including optimization.

RESUME

L'analyse de l'aptitude au service des systèmes porteurs des bâtiments implique de distinguer en général deux sortes d'incertitudes: le caractère aléatoire des variables de base et de la définition imprécise des états limites. Il est possible d'appliquer la théorie des ensembles flous au dernier type d'incertitude. Les critères en découlant, et définissant une aptitude au service défectueuse, permettent de formuler des concepts de dimensionnement probabilistes par l'application d'une méthode d'optimisation.

ZUSAMMENFASSUNG

Bei der Beurteilung der Gebrauchstauglichkeit von Gebäuden sind im allgemeinen zweierlei Unsicherheiten zu unterscheiden: die Zufälligkeit der Basisvariablen und die Unschärfe in der Definition der Grenzzustände. Letztere kann mit Methoden der Fuzzy-Set-Theorie behandelt werden. Daraus abgeleitete Kriterien mangelnder Gebrauchstauglichkeit erlauben die Formulierung wahrscheinlichkeitstheoretischer Bemessungskonzepte unter Einsatz von Optimierungsverfahren.

1. INTRODUCTION

Serviceability of building structures is their ability to perform adequately in normal use [1,2]. It is well recognized that due to several trends in modern design and construction, serviceability of building structures is becoming more and more important economic as well as technical issue [3,4,5,6]. Moreover, current procedure for dealing with serviceability are from various reasons insufficient and need to be improved.

One of the most important tasks is an identification of relevant functional requirements and their specification in terms of suitable set of serviceability parameters u_i. General guidance is offered in another contributions [7,8] at that colloquium. It appears that more requirements are often to be considered simultaneously, and both structural response to actions and deviations due to production procedures are to be considered. Nevertheless, in most cases only one serviceability parameter u is considered at a time (for example deflection at midspan, slope, amplitude, acceleration). In some cases, however, two or more parameters are to be investigated simultaneously (for example deflection and amplitude, amplitude and acceleration).

The most frequently applied serviceability criteria limit the actual values of serviceability parameter u_i, denoted $z_i(t)$, t being time, by time independent limiting values l_i [7]; in case of single parameter u the following inequality is traditionally used

$$z(t) \leq l \qquad (1)$$

This condition may be generalized for more complex quantities and/or a set of parameters u_i, actual values $z_i(t)$ and limiting values l_i. However, the fundamental question to be clarified first concerns rational and rigorous definition of the quantities entering any serviceability condition including the fundamental one, described by Equation (1).

2. UNCERTAINTIES

It is well recognized [6,9,10] that structural response $z(t)$ in Equation (1) depends on a number of basic variables of random nature such as actions, material properties and geometrical quantities. Consequently $z(t)$ is a random function of the time t, which may have considerable variability. Generally the structural response may be described by probability density function $\varphi_z(u,t)$, the mean function $\mu_z(t)$ and standard deviation function $\sigma_z(t)$, which become constants when structural response is described by time independent random variable z. In some cases probability distribution of structural response is not symmetrical and in that case skewness (likely to be positive) could be used [10].

The limiting value l on the right hand side of Equation (1) generally follows from functional requirements, which are often expressed in qualitative (verbal) way only and, consequently, are very subjective. Thus, the limiting values are also affected by considerable uncertainties, partly of a different nature than those involved in structural response $z(t)$. Evidently, in serviceability limit states in is rarely possible to distinguish unambiguously between acceptable and unacceptable state. This imprecision or vagueness in definition of limit states, appears to be the most significant source of great differences in evaluation and practical assessment of structural serviceability.

Evidently, there are two kinds of uncertainties to be considered when analyzing structural serviceability: randomness of basic variables or resulting variables and vagueness or imprecision in definition of serviceability limit states. While more familiar randomness of variables can be handled mathematically through the well established theory of probability, less familiar imprecision and vagueness in definition of serviceability limit states may be handled by methods of newly developing theory of fuzzy sets [11,12,13]. The following theoretical model for limiting values l_i of serviceability parameters is based on both concepts: randomness and fuzziness.

3. SERVICEABILITY LIMITS

Consider the fundamental case of a single serviceability parameter u (for example deflection at midspan-point or amplitude). It is assumed that with increasing parameter u, the ability of a structure to comply with specified functional requirements decreases and level of serviceability damage increases. In some cases a single distinct value l_0 could be identify, which separates unambiguously acceptable and unacceptable state. This, rather special case, may be described by stepwise membership function $\mu_S(u)$, shown in Figure 1. As a function of the serviceability parameter u it indicates membership of a structure in a set S of serviceable structures

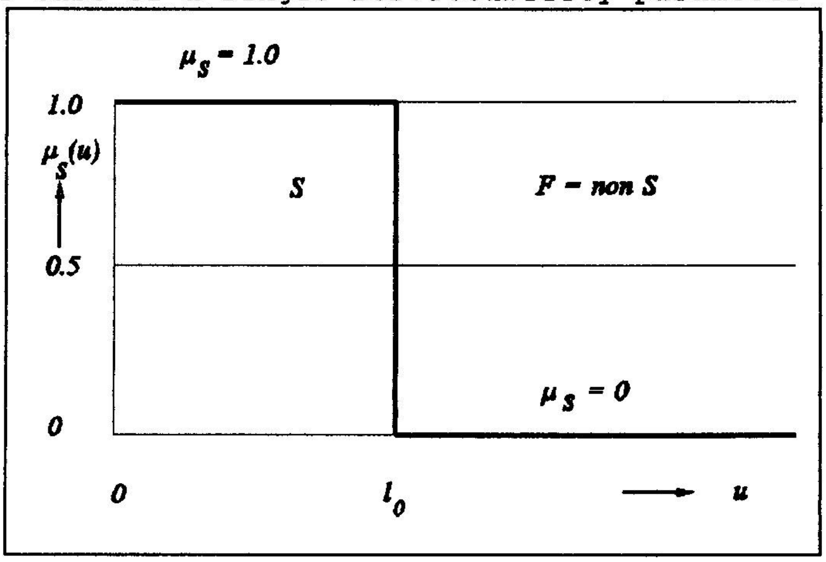

Figure 1 Membership function $\mu_S(u)$.

$$\mu_S(u) = 1 , \quad \text{if} \quad u < l_0 ,$$
$$\mu_S(u) = 0 , \quad \text{if} \quad u \geq l_0 , \tag{2}$$

Generally, however , the membership function $\mu_S(u)$ may be more complicated [14]. A conceivable and more realistic form of the function $\mu_S(u)$ could be

$$\mu_S(u) = 1 , \quad \text{if } u < l_1 ,$$
$$\mu_S(u) = \frac{(l_2 - u)^n}{(l_2 - l_1)^n} , \quad \text{if } l_1 \leq u < l_2 , \tag{3}$$
$$\mu_S(u) = 0 , \quad \text{if } l_2 \leq u ,$$

which is shown in Figure 2. Transition region, where the structure is gradually becoming unserviceable is specified by the lower limit l_1 and the upper limit l_2. Both these limits together with the exponent n characterize vagueness or fuzziness of the limit state and should be derived from its nature. Fuzzy set of unserviceable (damaged or failed) structures F is the complement of the set of serviceable structures S, thus F = *non* S. The membership function of the set F is given [11,13] as

$$\mu_F(u) = 1 - \mu_S(u). \tag{4}$$

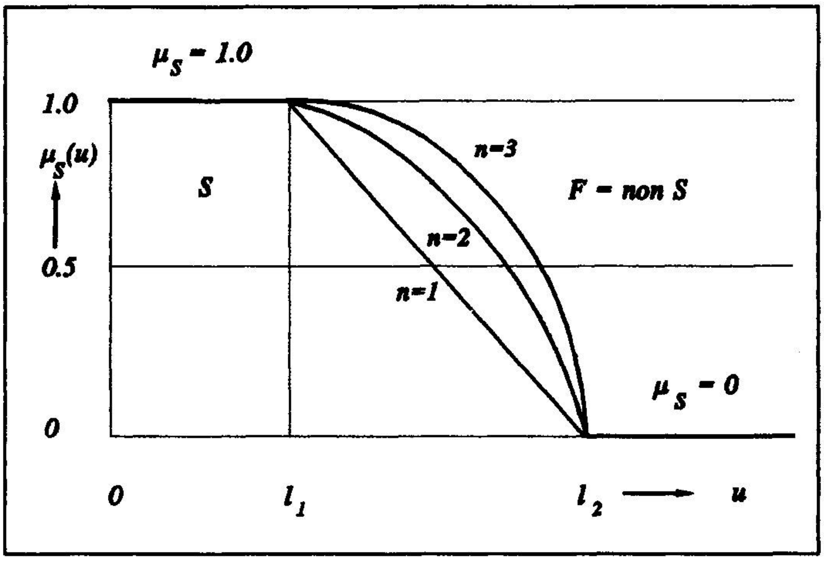

Figure 2 Membership function $\mu_s(u)$.

Furthermore, for a given serviceability level μ_s (function symbols without arguments are used to denote a variable or numerical value), serviceability parameter u (including both limits l_1 and l_2), may have considerable scatter. Similarly for a given parameter u, serviceability level μ_s may be a random variable. This randomness (not fuzziness) of membership function is caused by natural variability of human perceivability to various defects or due to random deviation in properties of installed machinery or secondary structures [6]. Therefore, the membership functions $\mu_s(u)$ and $\mu_F(u)$ are generally random functions of the serviceability parameter u. Variability of the membership function $\mu_F(u)$ for $n = 1$, which is the case used in the following analyses, is indicated in Figure 3. It is assumed, that above defined membership functions $\mu_F(u)$ represents the mean function and, furthermore, that for any given damage level μ_F, the probability density function of the serviceability parameter u may be described by normal distribution $\varphi_F(u'/\mu_F)$ having the mean equal to u'', for which $\mu_F(u'') = \mu_F$, and approximately constant (at least in a relevant interval of the parameter u) standard deviation σ_F.

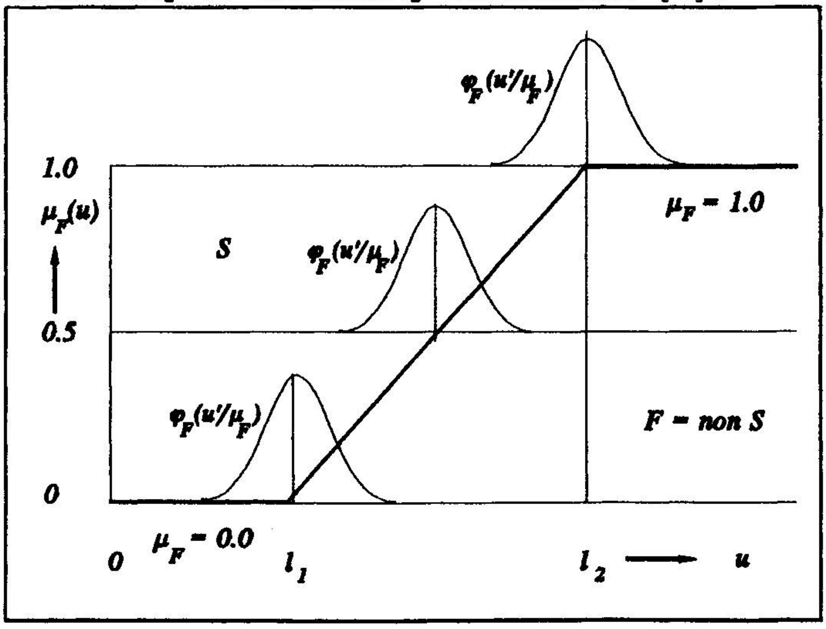

Figure 3 Membership function $\mu_F(u)$.

The above theoretical model of serviceability limits is consequently characterised by fuzziness characteristics l_l, l_t and the exponent n, and by the randomness characteristic represented by the standard deviation σ_f. Four extreme combinations of both concepts may be obviously recognised:

(a) deterministic case, when $l_l = l_t = l_0$, and $\sigma_f = 0$,
(b) pure fuzziness, when $l_l \neq l_t$ and $\sigma_f = 0$,
(c) pure randomness, when $l_l = l_t = l_0$ and $\sigma_f \neq 0$,
(d) fuzzy-random case, when $l_l \neq l_t$ and $\sigma_f \neq 0$.

From the most general combination of both concepts (d), which is treated bellow, the other combinations may be obtained by appropriate choice of the model characteristics. For example the case of pure randomness (c), which is considered in [6], is obtained for $l_l = l_t = l_0$.

4. UNSERVICEABILITY MEASURES

Expected unserviceability at a given damage level μ_f is the cumulative function $\Phi_f(u/\mu_f)$ of the serviceability parameter u,

$$\Phi_F(u/\mu_F) = \int_{-\infty}^{u} \varphi_F(u'/\mu_F)\, du'. \tag{5}$$

The total expected unserviceability (damage) corresponding to the serviceability parameter u is defined as weighted expected unserviceability with respect to all possible damage levels μ_f

$$\Phi_F(u) = \frac{1}{N} \int_0^1 \mu_F \Phi_F(u/\mu_F)\, d\mu_F\,, \tag{6}$$

where $N = 1/(n+1)$ is the normalizing factor to limit the total unserviceability into the interval $\langle 0,1 \rangle$. The limiting value l can be now defined as the parameter u for which the total expected unserviceability it is equal to a required value Φ_f, thus

$$\Phi_F(l) = \frac{1}{N} \int_0^1 \mu_F \Phi_F(l/\mu_F)\, d\mu_F = \Phi_F\,. \tag{7}$$

Taking into account random character of structural response $z(t)$, the probability of failure of a structure at a given damage level μ_f and time t is provided by the integral

$$p_f(\mu_F, t) = \int_{-\infty}^{\infty} \varphi_z(u/t) \Phi_F(u/\mu_F)\, du, \tag{8}$$

where $\varphi_z(u/t)$ denotes the probability density function of structural response $z(t)$. The total instantaneous unserviceability with respect to all possible damage levels μ_f at the time t is

$$p_f(t) = \frac{1}{N} \int_0^1 \mu_F\, p_f(\mu_F, t)\, d\mu_F\,, \tag{9}$$

The total probability of failure p_f within the whole intended life time T is then

$$p_f = \frac{1}{T} \int_0^T p_f(t)\, dt \,. \qquad (10)$$

The above unserviceability measures, given by Equations (7), (8), (9) and (10), can be used to formulate various types of design criteria.

Moreover, if the actual malfunction cost of any structure is proportional to the damage level μ_F, then the expected malfunction cost C_f can be expressed [16] as

$$C_f = \frac{1}{T} \int_0^T C_F(t)\, p_f(t)\, dt \approx p_f\, C_F \,. \qquad (11)$$

where the malfunction cost $C_F(t)$ due to the full unserviceability (when $\mu_F = 1$) is approximated by a time independent value C_F. If the total cost C could be expressed as the sum of the initial cost C_0 and expected cost C_f given by Equation (11), thus

$$C = C_0 + p_f\, C_F. \qquad (12)$$

then optimization procedure may be applied [15,16]. Necessary conditions for the minimum total cost follows from partial derivatives with respect to optimization variables.

5 EXAMPLE

The following example is based on experimental data [17], concerning serviceability limit state of visual disturbance. Excessive sags of *49* reinforced concrete floors and beams were recorded when annoying deformations were perceived. Observed disturbing sags z/L, where L denotes span of horizontal components [4], are within a broad range from *0.003* to *0.018*. Using this data the mean membership function $\mu_F(u)$, may be approximated by the tri-linear function ($n=1$), indicated in Figure 3. Further, the following fuz-

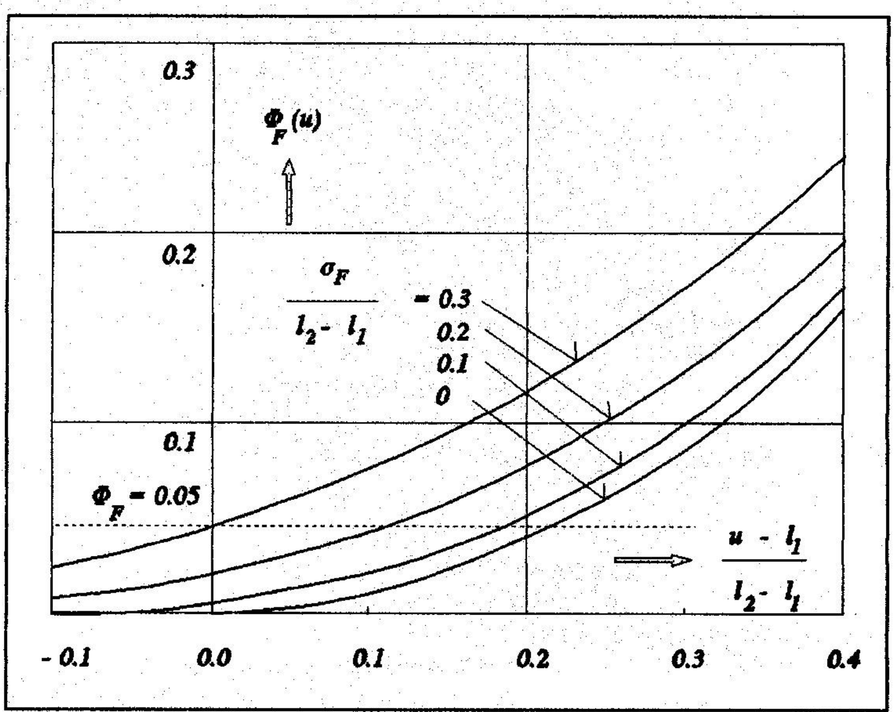

Figure 4 The function $\Phi_F(u)$ for $n = 1$.

ziness and randomness characteristics were derived from these data

$$\frac{l_1}{L} = 0.003, \quad \frac{l_2}{L} = 0.014, \quad \sigma_f = 0.05\,(l_1 - l_2). \tag{13}$$

The standard deviation σ_f was assessed from scatter of the data about the mean function as one twentieth of the transition length.

The total expected unserviceability $\Phi_f(u)$ for $n = 1$ is shown in Figure 4. It follows from Equation (7) and Figure 1, that for $\Phi_f = 0.05$ the limiting deflection is $l \approx l_l + 0.2\,(l_l - l_l) = L/192$, if $\Phi_f = 0.01$, then $l \approx l_l + 0.05\,(l_l - l_l) = L/282$. It should be however noted, that used experimental data do not include all the relevant information, and some additional assumptions were required to define the above model. More data, supplemented by information on level of observed damage, are urgently needed.

Let the cross section height h, be a single optimization variable. The sag z may be expressed as $z = K\,h^{-3}$, where K denotes a constant. If the initial cost $C_0(h)$ is proportional to h, then the first derivative of Equation (12) yield the condition [5]

$$\frac{C_0(h)}{C_F} = 3\mu_z \frac{\partial p(\mu_z, \sigma_z)}{\partial \mu_z} + 4\,\sigma_z \frac{\partial p(\mu_z, \sigma_z)}{\partial \sigma_z}. \tag{14}$$

The mean sag μ_z, determined for selected ratios C_f/C_0 and coefficients of variation σ_z/μ_z using Equation (14) and characteristics described by Equations (13) are given in Table 1.

Table 1. The optimum mean sag μ_z/L

Ratio σ_z/μ_z	Ratio C_f/C_0: 1	5	10	100	1000
0.00	1/159	1/251	1/282	1/391	1/498
0.05	1/181	1/316	1/376	1/571	1/781
0.10	1/220	1/431	1/532	1/855	1/1205
0.20	1/313	1/680	1/847	1/1351	1/2041

If the coefficient of variation $\sigma_z/\mu_z = 0.10$, then the optimum mean μ_z equals $L/220$ for $C_f/C_0=1$, $L/532$ for $C_f/C_0=10$. It appears, that commonly applied limiting values of the range from $L/360$ to $L/200$ correspond to relatively low cost of full malfunction C_f ($C_f/C0$ from 1 to 5) and high fuzzy probability of failure p_f (from 0.01 to 0.05). Consequently, commonly accepted serviceability constrains may be frequently uneconomical.

5. CONCLUSIONS

(1) Two kinds of uncertainties are to be distinguished when analyzing structural serviceability: randomness and vagueness.
(2) Imprecision and vagueness in definition of structural serviceability may be handled by methods of fuzzy set theory.
(3) Proposed unserviceability measures enable to formulate probabilistic concepts for design of structural serviceability

including optimization.

(4) Optimization of serviceability limit state due to visual disturbance indicates, that commonly used limiting values for sag of horizontal components may be uneconomical.

(5) Further research is recommended to concentrate on
- experimental data enabling more accurate theoretical models for vagueness in definition of limit states,
- fuzzy concept for multidimensional serviceability problems.

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