Evaluation of dynamic crowd effects for dance loads

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Evaluation des effets dynamiques sur les pistes de danse

Auswertung der dynamischen Mengeneffekte bei Tanzbelastung

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SUMMARY

When a crowd of people attempt the same repetitive movement, perfect synchronism is unlikely, and the total peak load produced by the group movement is less then the individual peak loads multiplied by the number in the group, i.e. the load is attenuated due to the crowd effect. This paper provides an estimation of this effect for dance loads. The loads for individuals and for an individual in a group of people are defined. The dynamic crowd factor is defined as the ratio of the maximum dynamic response for the two types of load. Three criteria are proposed and used to evaluate this factor. It is found that the factor is not affected by boundary conditions, dance frequency, structural frequency or damping.

RESUME

Lorsqu'une foule effectue les mêmes mouvements répétitifs, il est très rare d'arriver à un synchronisme total, et la charge maximale totale produite par les mouvements de groupe est inférieure à la charge maximale produite par chaque individu multipliée par le nombre de personnes du groupe; c'est-à-dire que la charge est atténuée par l'effet de groupe. Cet exposé donne une estimation de cet effet sur les charges des pistes de danse. Y sont définies la charge de chaque individu et la charge d'un individu dans un groupe de personnes. Le facteur dynamique de foule est le rapport de la réaction dynamique maximum des deux types de charge. On propose trois critères, qui sont utilisés pour évaluer ce facteur. Les calculs ont fait apparaître que le facteur n'est pas affecté par les conditions limites, le rythme de la danse, la fréquence structurelle ou l'amortissement.

ZUSAMMENFASSUNG

Wenn eine Menschenmenge dieselbe sich wiederholende Bewegung auszuführen versucht, ist perfekte Synchronisierung unwahrscheinlich, und die von der Gruppenbewegung erzeugte Gesamtspitzenlast liegt unter dem Wert, der sich aus der Multiplikation der Einzelspitzenlasten mit der Anzahl der Gruppenmitglieder ergibt; die Belastung wird also durch den Mengeneffekt gedämpft. Der Beitrag definiert die Belastung durch einen Einzeltänzer und für das Individuum in einer Menschengruppe. Der dynamische Mengenfaktor ergibt sich aus dem Verhältnis der maximalen dynamischen Reaktion beider Lasttypen. Für die Auswertung dieses Faktors werden drei Kriterien vorgeschlagen und eingesetzt. Es stellt sich heraus, dass der Faktor von Randbedingungen, Tanzfrequenz, Baufrequenz und Dämpfung nicht beeinflusst wird.

1 Introduction

When a crowd of people attempt the same repetitive movement, perfect synchronism is unlikely, even when there is a musical beat to follow. The effect of this lack of synchronisation is that the total peak load produced by group movement is less than the individual peak loads multiplied by the number in the group, *i.e.* the load is attenuated due to the crowd effect. This effect is important in evaluating overall crowd loads and their effect for design.

The loads considered in this paper are dynamic loads induced by human movement in time with a certain rhythm, such as dancing, jumping, swaying, stamping and aerobics. These loads are herein termed dance type loads. A full description of these loads consists of four components, namely load pattern, load frequency, load intensity and dynamic crowd effect. The first three items can be determined experimentally, but it is improbable that controlled experiments will be made with very large groups of people to enable the fourth parameter to be evaluated. Research has been conducted in this area on the first three items and the results can be found in both publications[1, 2, 9] and international guides[3, 6, 8]. However, much less information is available regarding to the dynamic crowd effect, despite the fact that it is potentially quite important.

Tilden[11] observed in tests with up to seven men on an ordinary platform scale that group movements could be random, and that perfect synchronism, to get the full effect of impulsive forces from a group of people, was improbable. A laboratory test was performed by Ebrahimpour and Sack[4]based on a 4ft by 8ft force platform where they measured dynamic loads of individuals and small groups. They noticed that individual loads could not be distinguished from the response of the group as a whole; thus the phase lag between individual peak loads could not be evaluated. This experimental investigation was further developed by the same authors using a 15ft by 12ft floor system, that was able to hold up to 40 people. The measured data indicated that the load amplitude per person reduces as the number of people involved increases[5]. Rainer, Pernica and Allen[10] also found that for groups of two, four or eight people jumping to a common timing signal, the peak values of dynamic load factors would be lower than that for single person. Ebrahimpour and Sack[4] suggested that an equivalent load-time history intensity per person in the crowd was determined by a simple averaging measure, *i.e.* adding M individual loads in which the random time phase lag is considered and then dividing by M and the area.

In this paper, a dynamic crowd factor is defined, which can be incorporated into existing load models to account for the reduction in the dynamic part of the load. The solution is developed based on a previous study[7]. A random phase lag, that obeys the normal distribution law, is introduced to represent the lack of synchronisation of individuals within a group of people. This is then used to obtain a statistical estimation of the reduction of loads due to the crowd effect. The study focuses on the simple dance activity, but the idea presented can be applied to more complex situations.

2 Basic Assumptions

The following assumptions are used in this study.

1. The floor is rectangular with uniform thickness and is either simply supported or clamped along its four edges.
2. The loads are described by existing models[2, 3, 7, 9] and are uniformly distributed on the floor, *i.e.*, loads are constant in the spatial domain but vary in the time domain.
3. When a group of people attempt the same repetitive movement, the phase lag, to describe the lack of coherence, is a random variable and obeys the normal probability law.

According to the first two assumptions, the response of a floor under dance type loads can be expressed analytically and only the fundamental mode needs to be considered in the analysis[7]. The third assumption establishes the random load model, albeit the distribution of the phase lag has to be defined.

3 Load Model for Crowd

3.1 Load model for individuals

An individual dancing or jumping load can be described analytically as follows[7]

$$F(t) = G\left[1.0 + \sum_{n=1}^{\infty} r_n sin(\frac{2n\pi}{T_p}t + \phi_n)\right] \tag{1}$$

where the coefficients r_n and ϕ_n are

$$r_n = \sqrt{a_n^2 + b_n^2} \qquad \phi_n = tan^{-1}\left(\frac{a_n}{b_n}\right)$$

$$\left.\begin{array}{lll} a_n = 0.5\left[\frac{1-cos(2n\alpha+1)\pi}{2n\alpha+1} + \frac{cos(2n\alpha-1)\pi-1}{2n\alpha-1}\right] & & \\ b_n = 0.5\left[\frac{sin(2n\alpha-1)\pi}{2n\alpha-1} - \frac{sin(2n\alpha+1)\pi}{2n\alpha+1}\right] & & \\ If \quad 2n\alpha = 1 & & n = 1,2,3,\ldots \\ then \quad a_n = 0 & & b_n = \pi/2 \end{array}\right\} \tag{2}$$

Where T_p and α are the load period and the contact ratio. The first part of the load G is the static load and indicates the individuals weight. The second part is the dynamic load induced by human movements.

For dancing, only the first Fourier term needs to be considered while for jumping and aerobics, the first three Fourier terms should be taken into account. According to the current analysis methods, Eq.(1) is applied to every individual in the crowd. This means that the dynamic crowd effect, *i.e.* the discrepancy in phase lag, is not considered.

3.2 Load model for crowd

To consider the dynamic crowd effect for a group of people, the individual load in a group is

$$F_s(t) = G\left[1.0 + \sum_{n=1}^{\infty} r_n sin(\frac{2n\pi}{T_p}t + \phi_n + \psi_s)\right] \qquad s = 1,2,...,M \tag{3}$$

Where M is the number of people involved. Comparing Eqs.1 and 3, the only difference is the phase lag ψ_s which characterises the difference between individuals and individuals within a group. When a group of people dance at a common frequency, it is reasonable to assume that the phase lag ψ_s is a random variable and obeys the normal distribution[12] as follows

$$\Psi = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-a)^2/2\sigma^2} \tag{4}$$

The parameter a corresponds to the central position and the parameter σ determines the spread. The area enclosed by the normal distribution Ψ (Eq. 4) and x equals one. It is worth noting that the area in the region [a-3σ, a+3σ] is 0.997. This states that it is probable that the random variable is located in this region. The phase lag ψ_s varies in the region [-π, π]. Therefore, the central position is zero, *i.e.*, $a = 0$. The standard deviation σ can be estimated by equating the above two regions $\sigma = \pi/3 = 1.0472 \doteq 1.0$. This is the typical value for σ adopted for the analysis. The selection of σ is central to the evaluation of the crowd effect. For a well co-ordinated group, may be a group of ballet

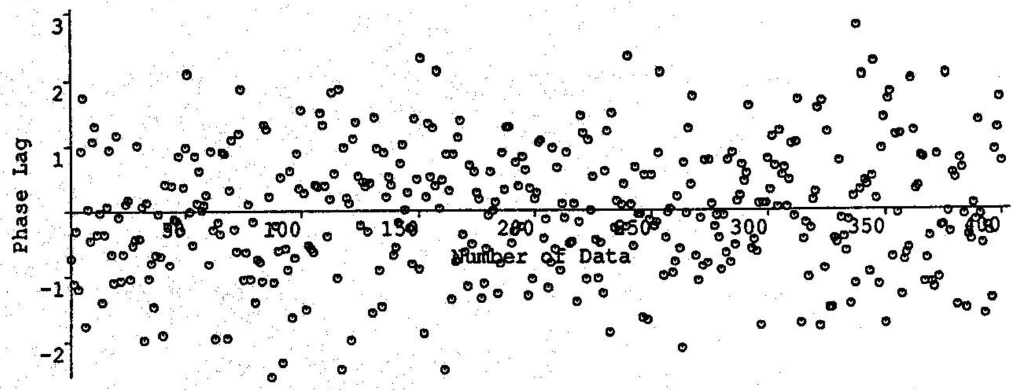

Fig.1 Computer Simulation of Random Phase Lag Obeying the Normal Distribution

dancers, σ is likely to be smaller than that for a group of disco dancers. However, for a large group, σ is likely to tend towards a specific value, which may depend on the *difficulty* of the dance; and also the difficulty of the dance may depend upon the dance frequency. Several cases will be evaluated, using various values of σ, to show the sensitivity of the crowd factor to the choice of σ.

The following summarises the method used to generate the M normally distibuted random angles in the range of $[-\pi, \pi]$.

1. For a normal distribution within the range $[-\pi, \pi]$, calculate the area within N equal bands. $(B_i, i = 1, 2, ..., N)$
2. Calculate the number of elements (MB_i) required in each band (to the nearest integer value).
3. Choose a phase angle randomly between $[-\pi, \pi]$ and accept the value if the sum of accepted values within its particular band is not greater than the total number required, otherwise discard.
4. Repeat 3 until all bands are complete.
5. Rearrange all the accepted elements in a random order to remove the effects of the selection process.

Fig.1 shows 400 generated random phase lags that follow the normal distribution.

4 Definitions of Dynamic Crowd Factor

Current methods for determining floor response are based on uniformly distributed dance loads neglecting the phase difference between individuals. It is suggested that a dynamic crowd factor can be incorporated into the current design method, as a simple modifier of the load. Several criteria can be used to define the dynamic crowd factor which is herein expressed by the symbol f_c with an additional subscript indicating the criterion adopted.

4.1 Displacement based criterion

The dynamic crowd factor is defined as the ratio of the maximum dynamic displacements induced by Eq.(3) and Eq.(1) respectively. *i.e.*

$$f_{cd} = \frac{A_{max}(\psi_s)}{A_{max}} \tag{5}$$

where A_{max} is the dynamic displacement at the centre of the floor and is the maximum absolute value during a period of steady state vibration. $A_{max}(\psi)$ has the same meaning but considers the phase discrepancy between individuals. The static displacement for both cases is the same and is not included in Eq. 5.

4.2 Acceleration based criterion

In a similar manner to the displacement based criterion, the dynamic crowd effect can be defined as follows

$$f_{ca} = \frac{\ddot{A}_{max}(\psi_s)}{\ddot{A}_{max}} \tag{6}$$

where $\ddot{A}_{max}$ indicates the acceleration at the centre of the floor and is the maximum absolute value during a period for a steady state vibration. $\ddot{A}_{max}(\psi_s)$ has the same meaning but considers the phase discrepancy between individuals.

4.3 Energy based criterion

This criterion can be represented by equating the sum of the work done by the dynamic part of the individual loads in a group (Eq.3) on the displacement $W(x,y)$ to that done by the corresponding part of the individual loads(Eq.1) multiplied by the dynamic crowd factor f_{ce}

$$f_{ce} = \frac{\sum_{s=1}^{M} W(x_s,y_s)F_{sd}(t)}{\sum_{s=1}^{M} W(x_s,y_s)F_d(t)} = \frac{\sum_{s=1}^{M} W(x_s,y_s)F_{sd}(t)}{F_d(t)\sum_{s=1}^{M} W(x_s,y_s)} \tag{7}$$

Where the displacement $W(x,y)$ could be the actual displacement or any feasible displacement that satisfies the boundary conditions.

For the first two criteria, the response of a floor should be determined in advance. For the third criterion the fundamental mode of a floor can be adopted instead of the actual response.

5 Expression of Dynamic Crowd Factor

The solution of the floor response can be determined for dancing and other human movements. However, only the dynamic crowd factors for dances, where only one Fourier term needs to be considered, are determined in this study.

5.1 Solution of floor response

A method of calculating floor vibrations induced by uniformly distributed human loads has been proposed by the authors[7]. Using this procedure, the central displacement of the floor, induced by a group of individual loads (Eq.3) can be obtained.

$$A_s(t) = \frac{G}{\omega_{11}^2 \iint mW_{11}^2(x,y)dxdy} \sum_{s=1}^{M} W_{11}(x_s,y_s) \left[1.0 + \sum_{n=1}^{\infty} \frac{r_n sin(nft - \theta_{ij} + \phi_n + \psi_s)}{\sqrt{(1-n^2\beta_{ij}^2)^2 + (2n\zeta\beta_{ij})^2}}\right] \tag{8}$$

The displacement $A_s(t)$ is at the centre of the floor and considers the individual's contribution to the group. Eq. 8 is derived based on the first two assumptions presented in the previous section. $W_{11}(x,y)$ is the fundamental mode of vibration of the floor and depends upon boundary conditions. This solution consists of two parts, the first is the static response due to the weight of the people, while the second is the dynamic response produced by human movements. When the discrepancy in phase ψ_s is neglected, Eq. 8 is reduced to the following form

$$A(t) = \frac{G\sum_{s=1}^{M} W_{11}(x_s,y_s)}{\omega_{11}^2 \iint mW_{11}^2(x,y)dxdy} \left[1.0 + \sum_{n=1}^{\infty} \frac{r_n sin(nft - \theta_{ij} + \phi_n)}{\sqrt{(1-n^2\beta_{ij}^2)^2 + (2n\zeta\beta_{ij})^2}}\right] \tag{9}$$

The corresponding accelerations can be obtained by double differentiation of the displacements (Eqs. 8, 9). The normalised modes for simply supported and clamped floors are respectively:

$$W_{11}(\xi,\eta) = sin(\xi)sin(\eta) \qquad 0 \le \xi \le 1, 0 \le \eta \le 1 \tag{10}$$

$$W_{11}(\xi,\eta) = (1-4\xi^2)^2(1-4\eta^2)^2 \qquad -0.5 \le \xi \le 0 - 0.5 \le \eta \le 0.5 \tag{11}$$
$$\xi = x/L_x \qquad \eta = y/L_y$$

where L_x and L_y are the dimensions of the floor.

5.2 Expression of three criteria

Taking the displacement based criterion (Eq. 5) and substituting Eqs(8, 9) into Eq. 5 gives.

$$f_{cd} = \frac{Max|\sum_{s=1}^{M} W(\xi_s,\eta_s)sin(nft-\theta_1+\phi_1+\psi_s)|}{Max|\sum_{s=1}^{M} W(\xi_s,\eta_s)sin(nft-\theta_1+\phi_1)|} \tag{12}$$

Since the phase lag ψ_s follows the normal distribution law, the maximum value in the time domain is at $nft - \theta + \phi = \pi/2$ for a steady state vibration. Therefore, the above equation can be reduced to

$$f_{cd} = \frac{\sum_{s=1}^{M} W(\xi_s,\eta_s)sin(0.5\pi+\psi_s)}{\sum_{s=1}^{M} W(\xi_s,\eta_s)} \tag{13}$$

Similar procedures can be applied to the acceleration and energy based criteria. Thus it is found that

$$f_c = f_{cd} = f_{ca} = f_{ce} \tag{14}$$

Eqs. 13 and 14 indicate that for dancing

1. The dynamic crowd factors are the same for either the displacement, the acceleration or the energy base criteria and can be represented in the simple form shown in Eq. 13.
2. The dynamic crowd effect is not related to the dance frequency, the floor frequency, the damping factor and the dead and live mass involved.

6 Determination of Dynamic Crowd Factor

Before using Eq. 13 to calculate the dynamic crowd factor, the following conditions should be considered

1. ψ_s describing the phase difference between the individual action and the music beat, is a random variable that obeys the normal distribution. The more random angles involved, the closer the actual distribution of these angles to the normal distribution. Therefore, a minimum number of 100 individuals in a group is used in this study.
2. Since ψ_s is a random variable, the dynamic crowd factor is correspondingly a random variable. To ensure a reliable result, 400 samples are calculated using Eq. 13, then statistical analysis is applied to determine the mean value and standard deviation of the dynamic crowd factor.

In the determination of the dynamic crowd factor, the individuals in a group are assumed to be uniformly distributed on a rectangle floor *i.e.* NX and NY people equally distribute in the x and y directions. The following data is adopted in the analysis

$$NX = 10, 15, 20, 25, 30, 35, 40, 45, 50$$

$$NY = 10, 15, 20, 25, 30, 35, 40, 45, 50$$

Hence there are in total 81 combinations to evaluate for each set of conditions. The boundary conditions considered are either the simply supported or fully clamped along the four edges of the floor. Also three values of σ (the standard deviation for the normal distribution) are given. Tables 1 lists the mean value of the dynamic crowd factors for selected cases evaluated for simply supported and clamped floors. Comparison of the dynamic crowd factors in these two tables shows

Table 1: Dynamic Crowd Factor

Simply Supported Boundary Condition									
	$\sigma = 0.9$			$\sigma = 1.0$			$\sigma = 1.1$		
	10	30	50	10	30	50	10	30	50
10	0.712	0.696	0.686	0.680	0.641	0.626	0.605	0.577	0.574
30	0.692	0.686	0.688	0.639	0.630	0.630	0.580	0.570	0.572
50	0.686	0.688	0.685	0.626	0.630	0.625	0.573	0.573	0.566
Clamped Boundary Condition									
	$\sigma = 0.9$			$\sigma = 1.0$			$\sigma = 1.1$		
	10	30	50	10	30	50	10	30	50
10	0.720	0.694	0.688	0.687	0.646	0.627	0.610	0.580	0.576
30	0.696	0.692	0.694	0.647	0.635	0.636	0.582	0.576	0.576
50	0.689	0.694	0.691	0.626	0.636	0.630	0.577	0.577	0.571

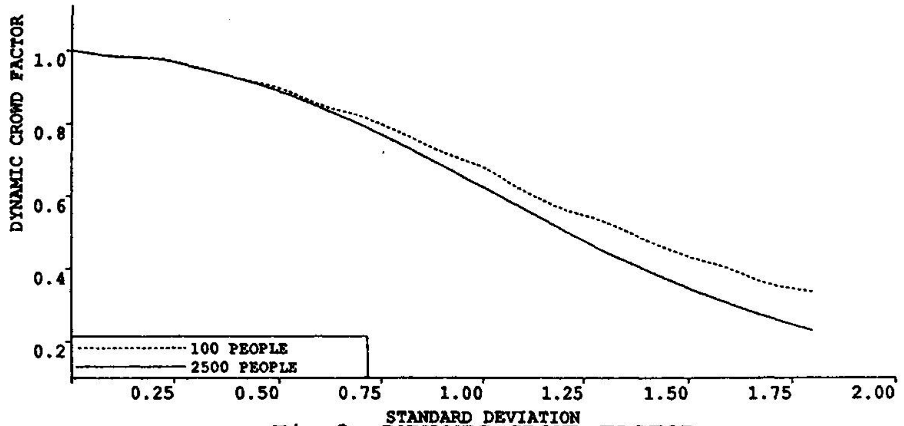

Fig.2 DYNAMIC CROWD FACTOR

1. The different boundary conditions basically do not affect the dynamic crowd factors.
2. The smaller the standard deviation (corresponding to that the distribution of the phase lag ψ_s), the higher the dynamic crowd factor.

Fig.2 gives the dynamic crowd factor f_c for a group of 100 and 2500 people and for the simply supported boundary condition as the function of the standard deviation σ. It has already been mentioned that this is central to the evaluation of the crowd effect and it's significance is shown in the figure. It is important not to overestimate σ as this would lead to an underestimation of the total load. Some experimental measurements had been made by Ebrahimpour and Sack[4], and these gave a value of about 0.55 for 40 people. This is slightly smaller than the values obtained using $\sigma = 1.0$ which are 0.68 for a group of 100 people and 0.63 for 2500 people. Hence, if the values for $\sigma = 1.0$ are adopted they are conservative with respect to the one measured value available.

7 Conclusions

In this study the dynamic crowd factor is defined which reflects the lack of synchronisation of individuals within a group. The conclusions drawn from the study for the dance activity are summarised as follows

1. The displacement, acceleration and energy based criteria give the same results.
2. The different boundary conditions basically do not affect the dynamic crowd factor

3. The dance frequency, structural frequency and damping factor do not affect the dynamic crowd factor.

4. For the proposed distribution of phase differences, the dynamic factor for a group of 100 people is 0.68 and for 2500 people it is 0.63.

The advantage of introducing the dynamic crowd factor for a group of people dancing is that it provides a better model of the loads and it also combines simply with existing analysis methods. Thus the load for a group of people and the response induced by the group of people are respectively

$$F(t) = G\left[1.0 + f_c \sum_{n=1}^{\infty} r_n sin(\frac{2n\pi}{T_p}t + \phi_n)\right] \quad (15)$$

$$A(x,y,t) = \frac{G \iint W_{11}(x,y)}{m\omega_{11}^2 \iint W_{11}^2(x,y)dxdy}\left[1.0 + f_c \sum_{n=1}^{\infty} \frac{r_n sin(nft - \theta_{ij} + \phi_n)}{\sqrt{(1-n^2\beta_{ij}^2)^2 + (2n\zeta\beta_{ij})^2}}\right] W_{11}(x,y) \quad (16)$$

Further analytical studies of the dynamic crowd effect to provide the dynamic crowd factor for other cases are required for

1. Activities where several Fourier coefficients are needed to model the loads.

2. Non-uniformly distributed load in which several modes need to be considered.

In addition, further experimental studies would be helpful.

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