Tuned mass dampers for continuous beams

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Amortisseurs de masse pour poutres continues

Abgestimmte Tilger für Durchlaufträger

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SUMMARY

Structural serviceability of buildings also includes comfort of occupants during structural response to dynamic excitation. Excessive dynamic response compromising comfort and serviceability can be suppressed by means of tuned mass dampers. Attention is focused upon the case, where this technique is used to suppress dynamic vibration of continuous beams often characterized by closely spaced frequencies and modes. A theory is presented in which modal analysis plays the key role. Analytical expressions for the response are presented and applied to an example, thereby highlighting some important observations to be accounted for in the tuning strategy.

RESUME

L'aptitude au service de bâtiments comprend aussi le confort humain dans le bâtiment sous excitation dynamique. L'atténuation des oscillations excessives se fait souvent avec des amortisseurs de masse. Le cas d'une poutre continue, pour laquelle les fréquences souvent sont très serrées, est résolu avec plusieurs amortisseurs de masse. Les auteurs présentent une théorie opérationelle basée sur une analyse modale. Cette analyse détermine la réponse structurelle et un exemple est présenté pour souligner certaines observations importantes dans le cas d'amortissement optimal.

ZUSAMMENFASSUNG

Die Gebrauchstauglichkeit von Gebäuden umfasst viele Komponenten, von denen sich eine auf den Komfort der Benutzer während der Reaktion des Gebäudes auf eine dynamische Anregung bezieht. Eine wohlbekannte Methode zur Unterdrückung übermässiger dynamischer Reaktionen besteht in der Anwendung von abgestimmten Tilgern. Hier wird insbesondere auf den Fall von Durchlaufträgern eingegangen, die oft durch eng beieinander liegende Frequenzen und Modalformen gekennzeichnet sind. Es wird eine Theorie dargestellt, bei der die Analyse der Schwingungsmoden eine Schlüsselrolle spielt. Es werden analytische Formeln für die Reaktion vorgestellt; anhand eines Beispiels werden einige wichtige Erfahrungen hervorgehoben, die bei der Strategie für die Abstimmung berücksichtigt werden sollten.

1. INTRODUCTION

Dynamic structural response is an important issue related to serviceability of buildings. The source of dynamic excitation is environmental loads such as wind load on tall buildings or occupant activities, installed equipment and machinery. Development of stronger materials enhance dynamic serviceability problems and remedial actions aiming at not exceeding limit values for human exposure to vibrations [1], [2] often is called for. Many of the efficient remedial actions belong to the categories of active or passive damping. Tuned mass dampers belonging to the second category have been used with success [2], [3].

The concept of tuning [2] is well known for ODOF systems. This paper presents a theoretical and operational basis for optimum tuning strategies for continuous beams for which tuning involves more modes. Modal analysis is applied. Operating in the complex plane, explicit determination of the modal response of the damped system as well as the response of the dampers are presented. Through examples application of the theory is elucidated.

2. MODAL ANALYSIS OF CONTINUOUS BEAMS

The well known modal theory for continuous beams is briefly outlined in the following.
Using well known symbols the free vibration problem for beams is governed by

$$EI\frac{\partial^4 W}{\partial x^4} + m\frac{\partial^2 W}{\partial t^2} = 0 \tag{1}$$

Modes F(x) and corresponding cyclic eigenfrequencies ω are determined by (2) in which non-dimensional quantities have been introduced

$$A[F] - \lambda B[F] = 0 \quad , A[\;] = \frac{d^4[\;]}{d\xi^4} \; , \; B[\;] = [\;] \; , \; \xi = \frac{x}{L} \; , \; \lambda = \frac{m\omega^2 L^4}{EI} \tag{2}$$

Operators A and B are selfadjoint and defining $<f,g>$ as the integration from 0 to 1 of $f(\xi)g(\xi)$

$$<A[F],G> = <A[G],F> \; , \; <B[F],G> = <F,G> = <G,F> = <B[G],F> \tag{3}$$

Therefore modes $F_i(\xi)$, $F_j(\xi)$ corresponding to distinct eigenvalues λ_i and λ_j are orthogonal i.e

$$<A[F_i],F_j> = <B[F_i],F_j> = \int_0^1 F_i(\xi)F_j(\xi)d\xi = 0 \quad for\; i \neq j \tag{4}$$

Furthermore we are free to normalise modes with respect to the operator B, therefore

$$<B[F_i],F_j> = <F_i,F_j> = \delta_{ij} \quad , \; <A[F_i],F_j]> = \lambda_j\delta_{ij} \tag{5}$$

To illustrate the phenomena of closely spaced eigenfrequencies of continuous beams two examples are shown on fig. 1 and 2. A simple and efficient technique has been used to identify the modes - based on Fourier expansion of $F(\xi)$ with special modelling at the supports, [4].

Adding a load term to (1), we obtain the equation for forced vibrations

$$EI\frac{\partial^4 W}{\partial x^4}+m\frac{\partial^2 W}{\partial t^2}=p(x,t) \tag{6}$$

Modal expansion of W in the orthonormalized system of modes $F_j(\xi)$

$$W(\xi,t)=\sum_{j=1} a_j(t)F_j(\xi) \tag{7}$$

combined with (5), (6) determines $a_j(t)$. Assuming harmonic excitation $p_j(\xi,t)=p(\xi)e^{i\omega t}$ and including modal damping ζ_j, the modal components a_j are determined by

$$\frac{d^2a_j}{dt^2}+2\zeta_j\omega_j\frac{da_j}{dt}+\omega_j^2a_j=p_j^0e^{i\omega t}\ ,\ p_j^0=\frac{1}{m}\int_0^1 p(\xi)F_j(\xi)d\xi \tag{8}$$

Introducing the well known frequency response function and phase angle of the j's mode $\Phi(\Omega_{(j)},\zeta_j)$ and $\varphi_j(\zeta_j,\Omega_{(j)})$ where $\Omega_{(j)}=\omega/\omega_j$ the response can be expressed :

$$W(\xi,t)=W(\xi)e^{i\omega t}\quad,\ W(\xi)=\sum_{j=1}\frac{1}{\omega_j^2}p_j^0\Phi(\Omega_{(j)})e^{i\varphi_j}F_j(\xi) \tag{9}$$

For ω sufficiently close to ω_i the response might become unacceptably high. Applying modal theory as used above, we extent the theory to include the effect of tuned mass dampers.

3. CONTINUOUS BEAMS WITH TUNED MASS DAMPERS

N_D mass dampers are placed at ξ_j, j=1, 2, ..N_D. The mass, damping and tuning frequency respectively is M_{Dj}, ξ_{Dj} and ω_{Dj}. The exiting frequency is ω and the response is determined by:

$$\frac{\partial^2 W}{\partial t^2}+\frac{EI}{L^4m}\frac{\partial^4 W}{\partial \xi^4}=\frac{1}{m}p(\xi)e^{i\omega t}-\sum_{k=1}^{k=N_D}\mu_k\frac{d^2y_k}{dt^2}\delta(\xi-\xi_k) \tag{10}$$

$$\frac{d^2y_j}{dt^2}+2\zeta_{Dj}\omega_{Dj}\left(\frac{dy_j}{dt}-\frac{\partial W}{\partial t}_{\xi=\xi_j}\right)+\omega_{Dj}^2(y_j-W_{\xi=\xi_j})=0 \tag{11}$$

δ is the Kronecker delta, $\mu_j=M_{Dj}/mL$ the mass ratio and y_j the movement of the j'th damper. Introducing (7), including N_m terms and using (5), - (10) and (11) take the form :

$$\frac{d^2a_j}{dt^2}+2\zeta_j\omega_j\frac{da_j}{dt}+\omega_j^2a_j=p_j^0e^{i\omega t}-\sum_{k=1}^{k=N_D}\mu_k\frac{d^2y_k}{dt^2}F_j(\xi_k)\quad j=1,2,..,N_m \tag{12}$$

$$\frac{d^2y_j}{dt^2}+2\zeta_{Dj}\omega_{Dj}\left(\frac{dy_j}{dt}-\sum_{k=1}^{k=N_m}\frac{da_k}{dt}F_k(\xi_j)\right)+\omega_{Dj}^2\left(y_j-\sum_{k=1}^{k=N_m}a_kF_k(\xi_j)\right)=0\quad ,\ j=1,2,...,N_D \tag{13}$$

We write the solution in the form (14) and make the variable transformation (15), (16):

$$a_j = a_j^0 e^{i(\omega t + \varphi_j)} \quad , \; y_j = y_j^0 e^{i(\omega t + \psi_j)} \tag{14}$$

$$j=1 \,..\, N_m \; : \quad z_{2j-1} = a_j^0 \cos\varphi_j \quad , \; z_{2j} = a_j^0 \sin\varphi_j \tag{15}$$

$$j=1 \,..\, N_D \; : \quad z_{2N_m+2j-1} = y_j^0 \cos\psi_j \quad , \; z_{2N_m+2j} = y_j^0 \sin\psi_j \tag{16}$$

(14) to (16) is introduced into (12) and (13) together with the definitions

$$F_{jk} = F_j(\xi_k) \quad , \; \Omega_{(Dj)} = \frac{\omega}{\omega_{(Dj)}} \qquad : j=1 \,..N_m \; , \; k=1 \,..N_D \tag{17}$$

whereby $N=2(N_m+N_D)$ equations for the N unknown z_i have been determined :

<u>$1 \leq j \leq N_m$</u>

$$(1-\Omega_{(j)}^2) z_{2j-1} - 2\zeta_j \Omega_{(j)} z_{2j} - \Omega_{(j)}^2 \sum_{k=1}^{k=N_D} \mu_k F_{jk} z_{2N_m+2k-1} = \frac{p_j^0}{\omega_j^2} \quad ...eq(2j-1)$$

$$2\zeta_j \Omega_{(j)} z_{2j-1} + (1-\Omega_{(j)}^2) z_{2j} - \Omega_{(j)}^2 \sum_{k=1}^{k=N_D} \mu_k F_{jk} z_{2N_m+2k} = 0 \quad ...eq(2j)$$

<u>$1 \leq j \leq N_D$</u>

$$-\sum_{k=1}^{k=N_m} F_{jk} z_{2k-1} + 2\zeta_{Dj} \Omega_{(Dj)} \sum_{k=1}^{k=N_m} F_{jk} z_{2k} + (1-\Omega_{(Dj)}^2) z_{2N_m+2j-1} - 2\zeta_{Dj} \Omega_{Dj)} z_{2N_m+2j} = 0 \quad ...eq(2N_m+2j-1)$$

$$-2\zeta_{Dj} \Omega_{(Dj)} \sum_{k=1}^{k=N_m} F_{kj} z_{2k-1} - \sum_{k=1}^{k=N_m} F_{kj} z_{2k} + 2\zeta_{Dj} \Omega_{(Dj)} z_{2N_m+2j-1} + (1-\Omega_{(Dj)}^2) z_{2N_m+2j} = 0 \quad ...eq(2N_m+2j)$$

$$\textit{That is}: \qquad K_{ij} z_j = b_i \tag{18}$$

after having solved these for z_i (note K_{ij} is <u>not</u> symmetric) we find for instance:

$$a_j^0 = sign(z_{2j-1}) \sqrt{z_{2j-1}^2 + z_{2j}^2} \quad , \; \varphi_j = Arctan\left(\frac{z_{2j}}{z_{2j-1}}\right) \quad : j=1 \,..\, N_m \tag{19}$$

and for the response (similar expression for the damper displacement) :

$$W(\xi,t)=W(\xi)e^{i\omega t} \quad , \quad W(\xi)=\sum_{j=1}^{j=N_m} a_j^0 e^{i\varphi_j} F_j(\xi) \tag{20}$$

3.1 Application, example

The beam shown in fig. 1 and 3 must be damped for exiting frequencies ω in an interval including the four lowest modes. The analysis includes 12 modes i.e. $N_m=12$. The number of dampers are 4, hence $N_D=4$ and N in (18) is $=2(12+4)=32$. Having determined the 12 modes according to the method briefly outlined in section 1, p_j^0 and the coefficients F_{jk} are determined. ω and the chosen tuning frequencies ω_{Di} determines $\Omega_{(j)}$ and $\Omega_{(Dj)}$. At this point - for each exiting frequency - K_{ij} and correspondingly all information regarding the response can be extracted from (18) to (20).
The tuning strategy must according to our experience observe the following rules: 1) Mass damper j, aiming at suppressing resonant response of mode j, should preferably be placed where mode no. j has it maximum value. 2) In addition to obeying rule no 1, care should be taken not to place dampers damping modes ω_i and ω_{i+1} close to each other as the dampers then will be working in opposite phase and cancel each other in the frequency range ω_i to ω_{i+1}. For instance if damper D2 is moved close to damper D1 and D4 is moved to a position near D2 we still obey rule no 1, but the damping arrangement would be very inefficient in the frequency range $\omega = 1.00$ to 1.15.

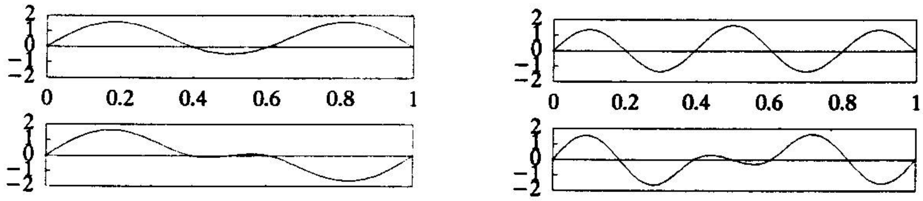

FIG. 1 Continuous beam over two internal supports. Length of span 1 and 3: 0.39474 L and of span 2 0.21052 L, where L is the total length. Shown from top to bottom - from left to right orthonormalized modes no. 1 to 4, which are lumped in pairs: $\sqrt{\lambda_i}$ = 76.8, 87.1 and 246, 288 for i=1 to 4. $\lambda_i = m\omega_i^2L^4/EI$.

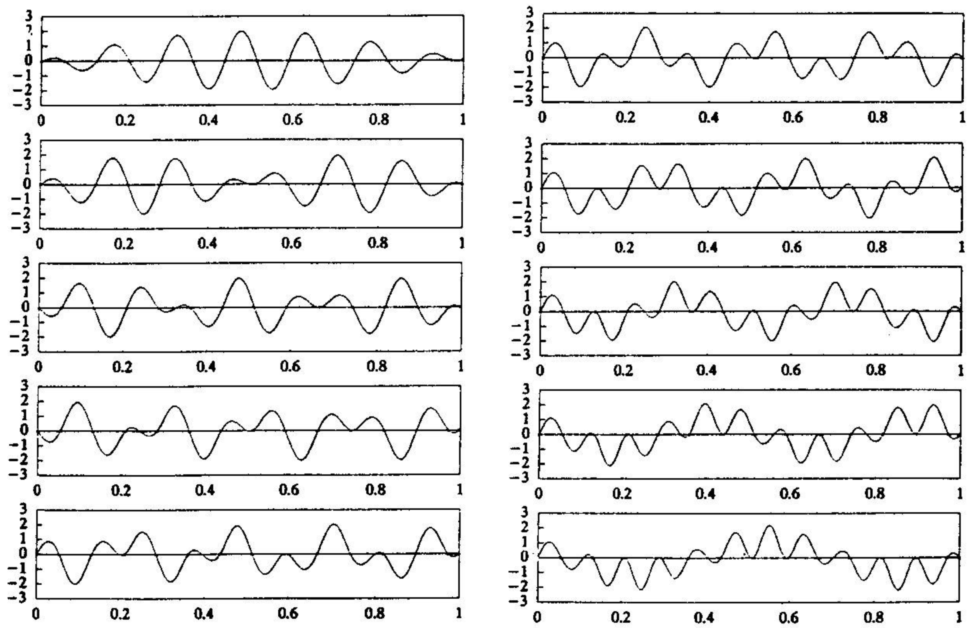

FIG. 2 Continuous beam over 13 internal supports. Length of span 1: 0.05536 L, length of span 2 to 13: 0.07631 L length of span 14: 0.028865 L. Shown from top to bottom from left to right orthonormalized modes 1 to 10. Eigenfrequencies only differ by a factor 2 from mode 1 to mode 10: $\sqrt{\lambda_i}$ = 1723, 1805, 1934, 2103, 2302, 2524, 2762, 3010, 3257 and 3492 for i=1 to 10. $\lambda_i = m\omega_i^2L^4/EI$

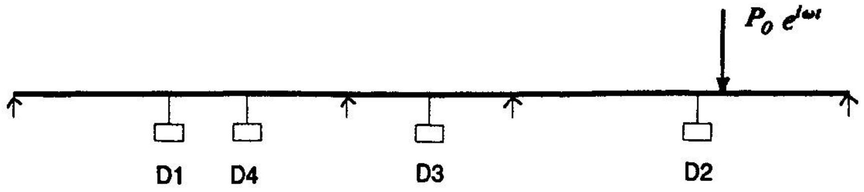

FIG. 3 Beam of fig.1. Single force acts at $\xi=0.85$ at exiting frequencies $0 \le \omega \le 4$. Four tuned mass dampers placed at $\xi_1=0.185$, $\xi_2=0.82$, $\xi_3=0.50$, $\xi_4=0.28$. 12 modes have been included in the analysis: ω_i, i=1 to 12: 1.000, 1.135, 3.202, 3.750, 5.016, 7.858, 8.397, 12.75, 14.08, 15.99, 21.28 and 22.16.Modal damping $\zeta_i=0.01$. The following tuning frequencies, mass ratio and damping values for the tuned mass dampers have been applied: $\omega_{Dj}=0.99$, 1.124, 3.17 and 3.713. $\mu_{Dj}=2\%$ and for ζ_{Dj} (j=1 to 4) two set of values have been used: 0.15 and 0.30.

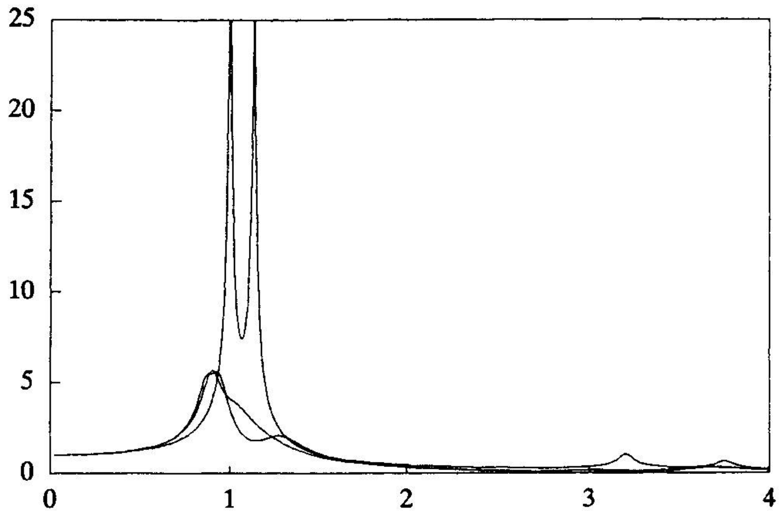

FIG. 4 Frequency response for the continuous beam of fig 1 and 3. x- axis frequency - y axis: the amplitude of the beam at the point where the load is applied. The two lower curves show the response for the damper configuration of fig 3 - data as given in legend to fig 2. The dampers reduce the peak values of the undamped system by a factor 5 to 10.

4. CONCLUDING REMARKS

A theory applicable to the case of dynamical exited beams being damped with tuned mass dampers has been presented. Based on the theory a user friendly edp based numerical tool enabling the designer to implement the optimum damping arrangement will be developed. The mass ratio to be applied to obtain efficient damping is very small (2% or less) - therefore the mechanical devices acting as tuned mass dampers can be very simple. Another potential avenue for future work of course will be to propose a simple design for such devices.

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