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## The required rotation capacity of joints in braced steel frames

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### Summary

One of the conditions for the use of plastic design for braced frames is that the joints have sufficient rotation capacity. In current practice, the requirements for rotation capacity are determined with the beam line theory. For a beam in an inner bay, this is reasonable; for a beam in an outer bay, this is questionable because of the deformations of the columns and difference in the joints.

In this paper an analytical model is presented for the determination of the required rotation capacity of joints. This model includes parameters like the resistance and stiffness of beams, columns and joints, as well as the second order effects in the columns.

The analytical model, however, is complicated for use in practice. Therefore, the beam line theory has been compared to this model, in order to investigate whether the beam line theory can be used safely for side spans. The results of a parameter study has shown that in certain cases, the beam line theory predicts too small (unsafe) values for the required rotation capacity compared to the analytical model. However, if the resistance of the joint connecting the beam to the outer column is smaller than half of the beam resistance, the ultimate load will not be reduced by more than 5% when the beam line theory is used. In other cases, a modification factor should be applied to the results of the beam line theory.

### List of symbols

$EI_{cln}$  is the stiffness of a column;  
 $EI_{bm}$  is the stiffness of a beam;  
 $f_{mod}$  is a modification factor;  
 $F_E$  is the Euler buckling load;  
 $k_m$  is a calculation factor;  
 $l_{bm}$  is the length of a beam;  
 $M_{Rd,bm}$  is the plastic moment capacity of a beam;



- $M_{Rd,cln}$  is the plastic moment capacity of a column;  
 $M_{Rd,i}$  is the moment capacity of a joint  $i$ ;  
 $M_{Rd,m}$  is the moment capacity of a joint to an inner column (mid joint);  
 $M_{Rd,s}$  is the smaller of the moment capacity of a joint to an outer column (side joint) and the resistance of an outer column for a moment transferred from the connected beam to the column;  
 $N_{sd}$  is the axial force in a column;  
 $P$  is the axial force acting at an outer column;  
 $q$  is the uniformly distributed load on a beam;  
 $S_{j,m}$  is the stiffness of a joint to an inner column;  
 $S_{j,s}$  is the stiffness of a joint to an outer column;  
 $S_s$  is the stiffness of a spring representing the behaviour of a joint to an outer column and this outer column;  
 $\alpha_1 l_{bm}$  is the length of a part of a column under a beam;  
 $\alpha_2 l_{bm}$  is the length of a part of a column above a beam;  
 $\beta$  is the relative stiffness;  
 $\zeta$  is a calculation factor;  
 $\nu$  is a multiplication factor for second order effects;  
 $\rho_m$  is the relative stiffness for a joint to an inner column;  
 $\rho_s$  is the relative stiffness for a joint to an outer column;  
 $\phi_{j,i}$  is the required rotation capacity of a joint  $i$ ;  
 $\phi_s$  is the required rotation capacity of a spring representing a joint to an outer column and this outer column;  
 $\phi_{j,m}$  is the required rotation capacity of a joint to an inner column;  
 $\phi_{j,s}$  is the required rotation capacity of a joint to an outer column.

## 1. Introduction

The response of steel frames is influenced by the resistance, stiffness and rotation capacity of the joints. Eurocode 3 Annex J [1] provides rules for the determination of the resistance and stiffness. For backgrounds to Eurocode 3 Annex J refer to [2, 3]. For the rotation capacity, only a set of "deemed to satisfy" rules is given. For example, it is stated that failure of a column web in shear will lead to sufficient rotation capacity. Joints not complying with these rules do not necessarily have insufficient rotational capacity in all cases. Then the verification should be based on a concept in which the available rotation capacity of a joint is compared with the required rotation capacity as determined in the frame analysis.

This paper reports the results of a study on the required rotation capacity in braced steel frames [4]. A review on required rotation capacity in braced frames was published by Bijlaard [5]. In the paper of Bijlaard the determination of the rotation capacity of joints was based on the beam line theory. The beam line theory assumes that the columns in the frame remain straight. This is of course questionable for outer columns, because their deformations (due to bending moments and second order effects) do have an influence on the response of the frame and thus the rotations in the joints.

This paper gives in section 2 a recollection of the beam line theory. In section 3, an

analytical model for the prediction of the required rotation capacity is presented. This analytical model takes into account the deformations of the outer column. In section 4, the analytical model is simplified by neglecting the influence of the column. It appears that in certain cases, predictions with the simple model are on the unsafe side. In section 5 the situations are determined where this unsafety is significant. In the last section, modification of the simple model is proposed.

## 2. Beam line theory

The beam line theory for the determination of rotation capacity in braced frames is based on the assumptions that:

- the columns remain straight;
- the two joints at both ends of the beam are identical;
- plastic hinges form in the joints.

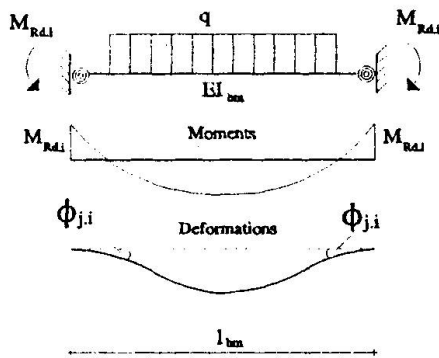


Fig. 1. Scheme for beam line

The system of the beam between two rotational springs is shown in fig. 1. The beam is loaded with a uniformly distributed load  $q$ . The required rotation capacity of the joints is:

$$\phi_{j,i} = \frac{ql_{bm}^3}{24EI_{bm}} - \frac{M_{Rd,i}l_{bm}}{2EI_{bm}} \quad (1)$$

## 3. The analytical model

The analytical model [4] is based on elastic-rigid plastic frame theory. This theory allows analytical treatment of the problem and represents the actual frame behaviour in a sufficient accurate way. The relation between the moment and the rotation of the joint is assumed to be bi-linear, see fig. 2. In this figure the angle  $S_j$  represents the stiffness of the joint,  $M_{Rd}$  the strength and  $\phi_{max}$  the maximum rotation. The moment curvature relation of beams and columns are also assumed to be bi-linear, see fig. 3.

A braced steel frame generally consists of storeys and bays. The beams in the outside bays are called side beams; the other beams are internal beams. For the analysis of the beams and joints, a sub frame is considered. This sub frame consists of a side beam and a part of an outer column: the length of the column is taken as two times half the storey height, see fig. 4. When the sub frame is at the first storey level, the length of the



column below the beam is equal to the storey height. The beam is loaded with a uniformly distributed load  $q$  and the column with an axial force  $P$ .

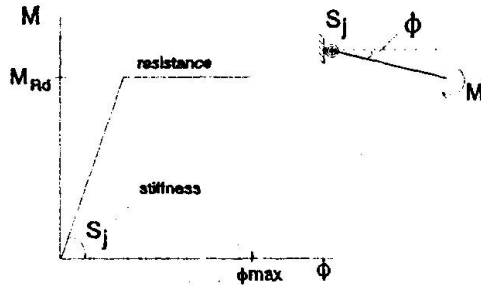


Fig. 2.  $M$ - $\phi$  diagram of a joint

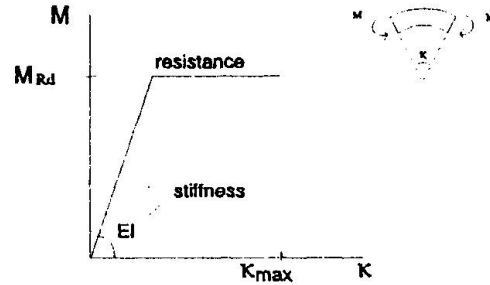


Fig. 3.  $M$ - $\kappa$  diagram of a member

This results in the scheme of the analytical model as given in fig. 5. The joint at the outer column is referred to as the side joint; the joint at the inner column as the mid joint.

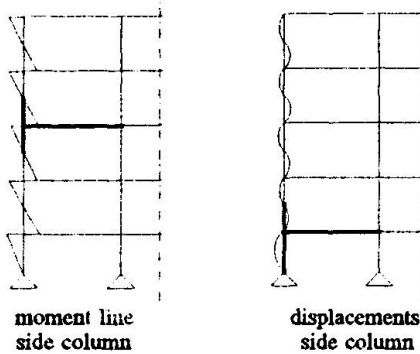


Fig. 4. Moments and displacements of the outer column

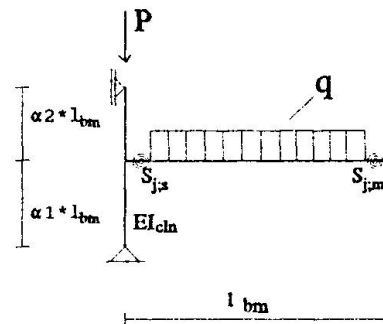


Fig. 5. Scheme of the analytical model

### 3.1 Beam behaviour

The behaviour of the beam in the sub frame can be analyzed with a modified beam line model as given in fig. 6. The side spring (on the left beam end) represents the behaviour of the outer column and the joint to this column in terms of resistance  $M_{Rd,s}$  and stiffness  $S_s$ . The spring on the right beam end represents the behaviour of the joint to the inner column (mid joint) in terms of resistance  $M_{Rd,m}$  and stiffness  $S_{j,m}$ .

The maximum uniformly distributed load  $q = 8 (M_{pl,bm} + 0,5 M_{Rd,m} + 0,5 M_{Rd,s}) / l_{bm}^2$  is reached when a full mechanism develops. A full mechanism develops when 'the last (third) plastic hinge' forms. At this stage the maximum rotation is reached in the springs. There are three different locations where a plastic hinge can form: in the two springs and in the span of the beam. So there are three possible 'end situations': the last plastic hinge forms in the side spring, in the mid joint or in the span of the beam. If a plastic hinge forms in the span of the beam, it is assumed that it forms exactly in the mid of the span. This gives only small errors in the prediction of the required rotational capacity of the joints, see [4].

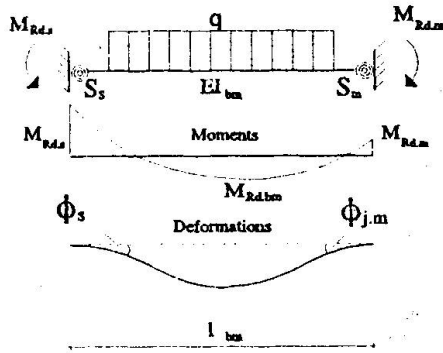


Fig. 6. Scheme for a beam in sub frame

The 'end situation' can be determined as follows. Assume that the last plastic hinge forms at mid span as shown in fig. 7.

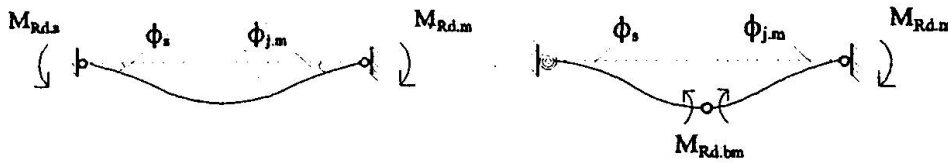


Fig. 7. Last hinge in span beam

Fig. 8. Last hinge in side spring

In that case, the rotations of the beam ends will exceed the elastic rotation of the side spring respectively the elastic rotation of the mid joint:

$$\phi_s \geq \frac{M_{Rd,s}}{S_s} \quad \text{and} \quad \phi_{j,m} \geq \frac{M_{Rd,m}}{S_m} \quad (2)$$

In analogy to equation 1, for the rotations in the beam near the joints can be written:

$$\phi_s = \frac{q l_{bm}^3}{24 EI_{bm}} - \frac{M_{Rd,s} l_{bm}}{3 EI_{bm}} - \frac{M_{Rd,m} l_{bm}}{6 EI_{bm}} = \frac{M_{Rd,bm} l_{bm}}{3 EI_{bm}} - \frac{M_{Rd,s} l_{bm}}{6 EI_{bm}} \quad (3)$$

$$\phi_{j,m} = \frac{q l_{bm}^3}{24 EI_{bm}} - \frac{M_{Rd,m} l_{bm}}{3 EI_{bm}} - \frac{M_{Rd,s} l_{bm}}{6 EI_{bm}} = \frac{M_{Rd,bm} l_{bm}}{3 EI_{bm}} - \frac{M_{Rd,m} l_{bm}}{6 EI_{bm}} \quad (4)$$

If the last plastic hinge does not form in the span of the beam, then we assume that the last plastic hinge forms in the side spring as shown in fig. 8. In that case for the mid joint should hold:  $\phi_{j,m} \geq M_{Rd,m} / S_{j,m}$ . The rotation of the mid joint is:

$$\phi_{j,m} = \frac{M_{Rd,s}}{S_s} - \frac{(M_{Rd,s} - M_{Rd,m}) l_{bm}}{6 EI_{bm}} \quad (5)$$

If the last plastic hinge doesn't form in the side spring, it can be concluded that it will form in the mid joint, as shown in fig. 9.

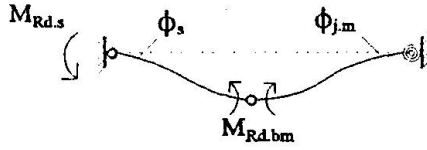


Fig. 9. Last hinge in mid joint

In that case, the rotation of the side spring is:

$$\phi_s = \frac{M_{Rd,m}}{S_{j,m}} - \frac{(M_{Rd,m} - M_{Rd,s})l_{bm}}{6EI_{bm}} \quad (6)$$

### 3.2 Behaviour of the outer column and the side joint

In paragraph 3.1 it was assumed that the side spring is bi-linear. In reality, the side spring should represent the behaviour of the outer column and the behaviour of the joint connecting the beam to the outer column.

The behaviour of the outer column is complex because of second order effects and the fact that one or two plastic hinges may form in this column. These hinges may form due to moments transferred from the beam to the outer column.

The rotation of side spring  $\phi_s$  is constituted from rotation of the column  $\phi_{cln}$  and the rotation in the side joint  $\phi_{j,s}$ . Conservatively, it is assumed that the rotation of the outer column is based on elastic behaviour ( $\phi_{cln} = M_{Rd,s} / S_{cln}$ ). In other words, if plasticity occurs in the column, the corresponding plastic rotations will be assigned to the side joint, so:

$$\phi_{j,s} = \phi_s - \frac{M_{Rd,s}}{S_s} \quad (7)$$

The moment capacity  $M_{Rd,s}$  of the side spring is the smaller of:

- the resistance of the outer column for bending moments transferred from the beam to the outer column and
- the resistance of the side joint.

The stiffness of the side spring is influenced by the stiffness of the side joint and the outer column as follows:

$$\frac{1}{S_s} = \frac{1}{S_{j,s}} + \frac{1}{S_{cln}} \quad (8)$$

For the stiffness of an outer column with pinned base can be written:

$$S_{cln} = \frac{\alpha_1 + \alpha_2}{\alpha_1 \alpha_2} \frac{3EI_{cln}}{l_{bm}} \quad (9)$$

The second order effects of the column can be taken into account by the multiplication factor  $n/(n-1)$ ,  $n = F_E / N_{sd}$ . The rotation in the column at floor level is then:

$$(\phi_{cln})^{\text{second order}} = \frac{n}{n-1} (\phi_{cln})^{\text{first order}} \quad (10)$$

Equations (9) and (10) result in:

$$\frac{1}{S_{cln}} = \frac{n}{n-1} \frac{\alpha_1 \alpha_2}{3(\alpha_1 + \alpha_2)} \frac{l_{bm}}{EI_{cln}} \quad (11)$$

### 3.3 Behaviour of the sub frame

Equations (2) to (11) give a complete description of the behaviour of the joints in the sub frame. The presentation of these equations can be improved by introducing:

$$\gamma = \frac{\alpha_1 \alpha_2}{3(\alpha_1 + \alpha_2)} \text{ for an outer column with pinned base} \quad (12)$$

$$\gamma = \frac{\alpha_1 \alpha_2}{4\alpha_2 + 3\alpha_1} \text{ for an outer column with rigid base} \quad (13)$$

$$\rho_s = \frac{S_{j,s} l_{bm}}{EI_{bm}} ; \quad \rho_m = \frac{S_{j,m} l_{bm}}{EI_{bm}} \quad (14)$$

$$v = \frac{n}{n-1} ; \quad n = \frac{F_E}{N_{sd}} ; \quad \beta = \frac{EI_{cln}}{EI_{bm}} \quad (15)$$

To determine where the last hinge forms, fig. 10 can be used. The rotations of the joints can be calculated by rewriting formulae (2) to (11). When the last plastic hinge forms in the span of the beam the rotations are as follows:

$$\phi_{j,s} = \frac{(2\beta M_{Rd,bm} - \beta M_{Rd,s} - 6v\gamma M_{Rd,s})l_{bm}}{6\beta EI_{bm}} \quad (16)$$

$$\phi_{j,m} = \frac{(2M_{Rd,bm} - M_{Rd,m})l_{bm}}{6EI_{bm}} \quad (17)$$

When the last plastic hinge forms in the side joint, the rotations are:

$$\phi_{j,s} = \frac{M_{Rd,s}}{S_{j,s}} ; \quad \phi_{j,m} = \frac{6(\beta + v\gamma\rho_s)l_{bm}M_{Rd,s} + \rho_s\beta(M_{Rd,s} - M_{Rd,m})l_{bm}}{6\beta\rho_s EI_{bm}} \quad (18)$$

When the last plastic hinge forms in the mid joint, the rotations are:

$$\phi_{j,s} = \frac{6\beta M_{Rd,m}l + \rho_m\beta(M_{Rd,m} - M_{Rd,s})l_{bm} - 6\rho_mv\gamma M_{Rd,s}l_{bm}}{6\rho_m\beta EI_{bm}} ; \quad \phi_{j,m} = \frac{M_{Rd,m}}{S_{j,m}} \quad (19)$$



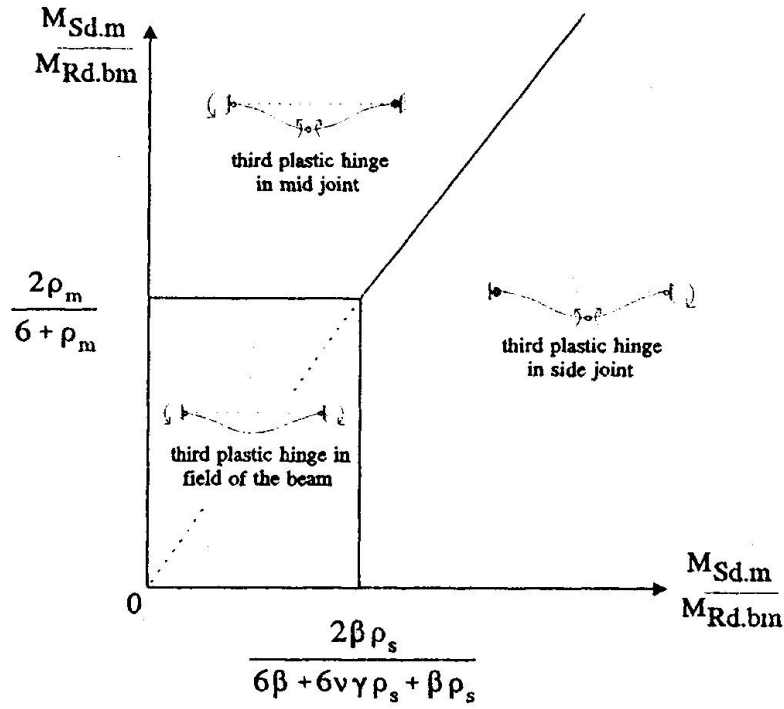


Fig. 10. Development of plastic hinges in the sub frame

#### 4. Simplification

With the analytical model accurate values of the required rotation capacity can be determined. However, the model is complicated for use by practitioners. Therefore, simplification is necessary. To simplify the analytical model, the outer column's influence is removed from this model by assuming that this column remains straight. This results in a simple model, i.e. a beam between two straight columns, with two different springs in strength and stiffness. The simple model is identical to the model of figure 6, but  $S_s = S_{j,s}$  and  $\phi_s = \phi_{j,s}$ .

With the simple model, the required rotation capacity can be predicted as follows. When equation (20) is fulfilled, then the last plastic hinge forms in the mid span of the beam. In that case, the rotation capacities of the side joint  $\phi_{j,s}$  and the mid joint  $\phi_{j,m}$  can be determined with equations (3) and (4). Otherwise, when equation (21) is fulfilled, then the last plastic hinge forms in the side joint. The rotation capacity of the mid joint  $\phi_{j,m}$  can be determined with equation (5). When the last hinge forms in the mid joint, the required rotation capacity for the side joint  $\phi_{j,s}$  can be determined with equation (6).

$$\frac{M_{Rd,s}}{M_{Rd,bm}} \leq \frac{2\rho_s}{6 + \rho_s} \quad \wedge \quad \frac{M_{Rd,m}}{M_{Rd,bm}} \leq \frac{2\rho_m}{6 + \rho_m} \quad (20)$$

$$\frac{M_{Rd,m}}{M_{Rd,s}} \leq \frac{\rho_m(6 + \rho_s)}{\rho_s(6 + \rho_m)} \quad (21)$$

## 5. Comparison of the two models

The two models described sections 3 and 4 have been compared. The simple model ideally should predict the required rotation capacity as calculated with the analytical model, or a higher value of this rotation capacity. Actually the analytical model is the same as the simple model, only the outer column has more flexibility. In the analytical model the column takes part in the rotation of the beam's end. In that case, the required rotation capacity of the side joint is smaller than in the case of a straight column (the simple model). Thus the simple model always predicts too large rotations for the side joint.

For the mid joint holds: the stronger the side joint and the more flexible the outer column is, the later the last plastic hinge forms in the side joint. In this case, the simple model, which already would have formed the last plastic hinge, calculates too small rotations  $\phi_{j,m}$ .

When the rotation capacity in the mid joint  $\phi_{j,m}$ , as determined by the simple model, is smaller than the rotation  $\phi_{j,m}$  found with equation (18), the plastic mechanism in the beam will not be fully reached and the uniformly distributed load  $q$  will be lower than  $8 (M_{Rd,bm} + 0,5 M_{Rd,s} + 0,5 M_{Rd,m}) / l_{bm}^2$ . In that case,  $q$  at failure can be expressed as a function of  $\phi_{j,m}$ :

$$q_{max} = \frac{24k_m EI}{(6+k_m)l_{bm}^2} * \phi_{j,m} + \frac{24(2M_{Rd,bm} + M_{Rd,m} + 8k_m(M_{Rd,bm} + M_{Rd,m}))}{(6+k_m)l_{bm}^2} \quad (22)$$

with:

$$k_m = \frac{\beta \rho_m}{\beta + \nu \gamma \rho_s} \quad (23)$$

In equation (22), the rotation in the mid joint  $\phi_{j,m}$  should be determined by the simple model.

It is assumed that a reduction of  $q$  (error on the unsafe side) of 5% is acceptable. By means of a parameter study, the situations were determined when the error exceeded 5%.

## 6. Modification factor

From the parameter study it appeared that the reduction of  $q$  is never exceeding 5% when the strength of the side joint is smaller than  $0.5 M_{pl,bm}$ . When the strength of a side joint is more than  $0.5 M_{pl,bm}$ , the required rotation capacity, as found with the simple model, has to be multiplied with a modification factor. For the determination of this modification factor it is referred to [4]. This modification factor is based on upper bounds of the second order factor  $\nu$  and the geometry factor  $\gamma$ , and is equal to:

$$f_{mod} = \left( \frac{6}{\rho_s} + \frac{1}{\beta} + 1 \right) \frac{M_{Rd,s}}{M_{Rd,bm}} - 1 \geq 1 \quad (24)$$

The parameter study also showed that in the majority of cases the last plastic hinge forms



in the span of the beam. The reduction of  $q$  has also been investigated, by assuming that in the simple model the last hinge always forms in the beam span. Further, the reduction factor has been applied in case the strength of the side joint is more than half the beam resistance. It proved for the frames investigated, that the reduction of  $q$  is never more than 5%.

## 7. Conclusions

By adopting elastic-rigid plastic frame behaviour, the required rotation capacity of joints in the outer bays of braced steel frames can be determined analytically. To make this analytical model suitable for use in practice, it can be simplified to a 'modified beam line model'. In this model is assumed that the outer columns remain straight. The value of the rotation capacity according to the simple model has to be multiplied with a modification factor in case the resistance of the joint to the outer column exceeds half of the beam resistance. This 'modified beam line model' can be written as:

$$\phi_{j,s} = \frac{(2M_{Rd,bm} - M_{Rd,s})l_{bm}}{6EI_{bm}} ; \quad \phi_{j,m} = \frac{(2M_{Rd,bm} - M_{Rd,m})l_{bm}}{6EI_{bm}} \cdot f_{mod}$$

In case  $M_{Rd,s} \leq 0.5 \cdot M_{Rd,bm}$ ,  $f_{mod} = 1$ , otherwise:

$$f_{mod} = \left\{ \frac{6EI_{bm}}{S_{j,s}l_{bm}} + \frac{EI_{bm}}{EI_{cln}} + 1 \right\} \cdot \frac{M_{Rd,s}}{M_{Rd,bm}} - 1 \geq 1$$

## References

- [1] EUROCODE 3, ENV - 1993-1-1, *Revised annex J*, Design of Steel Structures, CEN, European Committee for Standardization, Document CEN / TC 250 / SC 3 - N 419 E, Brussels, Belgium, June 1994.
- [2] Zoetemeijer P., *Summary of the research to bolted end plate connections*, Report 6-85-7, University of Technology, Delft, the Netherlands, 1985.
- [3] Jaspart, J.P., *Etude de la semi-rigidite des noeuds poutre-colonne et son influence sur la resistance et la stabilite des ossatures en acier*, University of Liège, Belgium, 1991.
- [4] Boender E.H., *Study to required rotation capacity of joints in braced steel frames (in Dutch)*, report 95-CON-R0879, University of Technology and TNO Building and Construction Research, Delft, the Netherlands, 1995
- [5] Bijlaard F.S.K., *Requirements for Welded and Bolted Beam-to-Column Connections in Non-Sway Frames*, Joints in Structural Steelwork, edited by J.H. Howlet W.M. Jenkins and R. Stainsby, Pentech Press, London, United Kingdom, 1981