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Probability-Based Optimisation of Inspection Intervals for Steel Bridges

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Summary

The fatigue safety of steel bridges is achieved through design of individual components and inspections with subsequent repair of detected cracks. Each safety item has a certain cost and it is of importance to minimise the total expected cost for the lifetime of the component. The optimisation parameters are the inspection times, but other variables (material properties, inspection qualities,...) can be also introduced. Two optimisation problems are treated in this paper. The first one concerns the optimisation of the next inspection time, while the second one treats of the optimisation of the regular inspection interval during the component lifetime. An example of a welded joint highlights the different concepts. It has to be noticed that the techniques presented in the paper are not restricted to fatigue problems, but can be applied to a wide variety of deterioration phenomena.

1. Introduction

Maintenance and rehabilitation of existing structures has become of great concern for public or private owners during the last decades. All the structures made by men are time-degrading because of phenomena such as corrosion, fatigue, erosion,... induced sometimes by poor durability design, lack of quality control or absence of regular inspection and maintenance actions.

Budgets for maintenance and rehabilitation are always limited. In order to rationalise maintenance actions, management systems have been developed, helping to a standardisation of the procedures through the development of inspection manuals and the implementation of databases. Experience acquired with these procedures leads today to define other approaches in which rationality is based on the optimisation of maintenance costs. This optimisation requires methods which take into account technical, economical, management points of view as well as theoretical or practical aspects. Offshore engineering has already successfully rationalised its maintenance actions (see for instance [1], [2]) by using probabilistic concepts. The present paper attempts to illustrate such an approach in the field of steel bridges with respect to the problem of welded joints damaged by fatigue. In welded joints, the cracks are often localised at the weld. The welds induce some defects which help small cracks to appear. They are growing under loading and can lead to the joint failure. The conditions governing crack growth propagation are numerous, and in general, random. Therefore, an appropriate analysis of fatigue phenomena consists by treating the problem in a probabilistic manner. But, the probabilistic model must be flexible enough to include inspection results with their qualities for assessing damage in a better manner. Such an approach must help to consider all the events (inspections, repairs, failure) which can occur during the conventional lifetime of the joint. As costs can be linked to these events, it is then possible to build an optimisation procedure aiming to minimise the total maintenance cost with respect to conventional reliability degrees.

2. Models

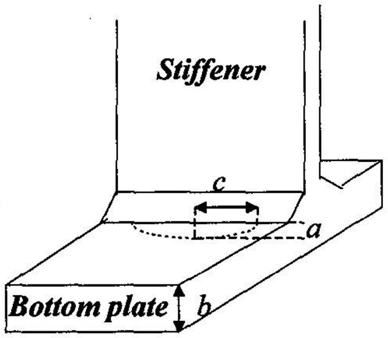

Fig.1 Crack details in a welded joint

Two types of model are used to determine a management strategy: event models and maintenance models. The event models are mathematical models which describe events occurring in the component. For welded joints, these models contain the fatigue crack growth model and the detection model. These mathematical models permit to assess the corresponding probabilities of events occurrence. The maintenance models are linked to the strategy of maintenance and management. In these models, the events probabilities determined with the event models are combined with costs for providing total expected maintenance costs.

2.1. Crack growth model

The model used in this paper is the Paris law corresponding to the opening of a semi-elliptical crack in the bottom plate of a « stiffener-bottom plate » welded joint (Figure 1)

$$\frac{da}{dN} = C(\Delta K)^m = C\left[Y(a,c)M_k(a)\Delta S\sqrt{\pi a}\right]^m \tag{1}$$

where

- a is the crack size, c half the crack length, b the bottom plate thickness,
- N is the number of cycles, ΔK the stress intensity factor range, ΔS the stress range,
- $Y(a, c)$ is the stress intensity geometry correction factor, $M_k(a)$ the stress intensity concentration correction factor, C and m two material parameters.

Under the threshold stress intensity range ΔK_{th}, the crack does not grow. Equation (1) does not distinguish damaging and non damaging cycles. The modified model of reference [3] introduces a correction function $G(a, c)$ which allows this discrimination:

$$\frac{da}{dN} = C\left(Y(a,c)M_k(a)\sqrt{\pi a}\right)^m G(a,c)\left(\frac{2E(\Delta S)}{\sqrt{\pi}}\right)^m \Gamma\left(1+\frac{m}{2}\right) \tag{2}$$

where

$$G(a,c) = 1 - \frac{1}{\Gamma\left(1+\frac{m}{2}\right)}\gamma\left(1+\frac{m}{2};\frac{1}{2}\frac{(\Delta K_{th})^2}{\left(\sqrt{\frac{2}{\pi}}E(\Delta S)Y(a,c)M_k(a)\sqrt{\pi a}\right)^2}\right) \tag{3}$$

$E(\Delta S)$ is the mean of the stress range process. $\Gamma(.)$ and $\gamma(.;.)$ are respectively the complete and incomplete Gamma functions. $N(t) = \nu_0 t$ is the number of cycles at time t. Equations (2) and (3) have been obtained under the assumption that the stress ranges follows Rayleigh distributions and by using the equivalent stress range approach [4].

A safety margin expresses the frontier between damage and non damage. A straightforward safety margin for fatigue reliability assessment can be defined by $Z(t) = a(t) - a_c$, where a_c is the critical crack size, which can be chosen to a conventional value or according to a fracture criterion. $a(t)$ is the crack size at time t after $N(t)$ cycles. The integration of Equation (2) between an initial crack size a_0 and a_c is equivalent to another safety margin :

$$M(t) = \int_{a_0}^{a_c} \frac{dx}{\left[Y(a,c)M_k(a)\sqrt{\pi a}\right]^m G(a,c)} - CN(t)\left(\frac{2E(\Delta S)}{\sqrt{\pi}}\right)^m \Gamma\left(1+\frac{m}{2}\right) \quad (4)$$

2.2. Detection model

A measurement system cannot detect a crack when it is too small. A threshold value exists and corresponds to the detection level under which detection is no longer reliable. This threshold crack size, a_d is the smallest detectable crack size allowable by the measurement system. The probability to detect a crack depends on a_d, and on the precision of the system. In general, a_d is never precisely known, and therefore, has to be considered as a random variable. The detection probability is consequently the probability that the crack size is greater than a_d. If x is the crack size and $F(.)$ the distribution function for a_d, the probability to detect a crack is then:

$$P_d(x) = P(x \geq a_d) = F(x) \quad (5)$$

The knowledge of $P_d(x)$ is therefore sufficient for determining the distribution function of a_d. Several models have been developed for explaining the uncertainties met by using Non Destructive Inspection techniques [4]. For instance, the ultrasonic detection method leads to a detection level a_d which can be modelled by a lognormal distribution.

2.3 Events margins

Qualitative or quantitative information can be given by inspections. Each of these results is an event, associated to an event margin H and to an occurrence probability.

Qualitative inspection results are information upon the detection or the non detection of an event related to a particular phenomenon. The information is expressed by:

$$H \leq 0 \quad (6)$$

For fatigue crack growth propagation, the non-detection of a crack size A_i after N_i cycles corresponds to an event where the crack size $a(N_i)$ is *smaller than* A_i. The no detection event is expressed by

$$H_{nd}(a_i) = -\int_{a_0}^{A_i} \frac{dx}{\left[Y(a,c)M_k(a)\sqrt{\pi a}\right]^m G(a,c)} + CN_i\left(\frac{2E(\Delta S)}{\sqrt{\pi}}\right)^m \Gamma\left(1+\frac{m}{2}\right) \leq 0 \quad (7)$$

The detection event is the complementary of the previous one and the event is then:

$$H_d = \int_{a_0}^{A_i} \frac{dx}{\left[Y(a,c)M_k(a)\sqrt{\pi a}\right]^m G(a,c)} - CN_i\left(\frac{2E(\Delta S)}{\sqrt{\pi}}\right)^m \Gamma\left(1+\frac{m}{2}\right) \leq 0 \quad (8)$$

Quantitative inspection results correspond to measurements of an event related to a particular phenomenon. The information is expressed by:

$$H = 0 \tag{9}$$

For fatigue crack growth propagation, the detection with measurement corresponds to an event where the crack $a(N_i)$ after N_i cycles, is *equal* to A_i, measurement at time t_i. The « detection with measurement » event is expressed by

$$H = \int_{a_0}^{A_i} \frac{dx}{\left[Y(a,c)M_k(a)\sqrt{\pi a}\right]^m G(a,c)} - CN_i\left(\frac{2E(\Delta S)}{\sqrt{\pi}}\right)^m \Gamma\left(1+\frac{m}{2}\right) = 0 \tag{10}$$

2.4 Maintenance model

The maintenance model which has been used, is the conditional maintenance with regular inspection interval. A conditional maintenance requires to define criteria or conditions according to which maintenance actions will be engaged. This has to be made by quantifying crack sizes by a detection method and by ranging crack sizes in a finite number of severity classes.

Let us assume that this finite number of classes is limited to (n_R+1) where the first class corresponds to non detectable cracks. Let us call this class $I_0 = [0, a_d[$ where a_d corresponds to the smallest crack size which can be detected. If there are n_R types of possible repairs, then it follows that any crack with size $a \in I_i = [a_{i-1}, a_i[$ is repaired by the repair technique N.i. Let us note that the decision interval $I_1 = [a_d, a_1[$ can correspond to a detection followed by no repair actions. Figure 2 illustrates the different scenarios occurring at each inspection time according to a conditional maintenance strategy. T_s is a reference period which can be the next inspection time or the conventional lifetime T_f. The inspection events are therefore defined as it follows:

- the event corresponding to a non detection is $\left\{H^0 \le 0\right\}$,
- the event corresponding to a repair method N.i is $\left\{H^i \le 0\right\}$.

In fact, the events have to be rigorously written:

- $\left\{H^0 \le 0\right\} = \left\{H_{nd}(a_d) \le 0\right\}$,
- $\left\{H^1 \le 0\right\} = \left\{H_d(a_d) \le 0 \cap H_{nd}(a_1) \le 0\right\}$
- $\left\{H^i \le 0\right\} = \left\{H_d(a_{i-1}) \le 0 \cap H_{nd}(a_i) \le 0\right\}$ for $i = 2, \cdots, n_R - 1$
- $\left\{H^{n_R} \le 0\right\} = \left\{H_d\left(a_{n_R-1}\right) \le 0\right\}$.

Nevertheless, for the sake of simplicity, the first notation will be kept in the following developments. Each action at each inspection has an effect on the event and safety margins at the next inspection time. It is therefore necessary to introduce another notation for describing the events sequences. For instance, with an action k at time t_1 and an action l at time t_2, the safety margin at time $t_2 \le t \le t_3$ will be denoted $M^{k,l}$ and the event margin at time t_2 will be denoted $H^{k,l}$. $M(t)$ will still define the safety margin before the first inspection time.

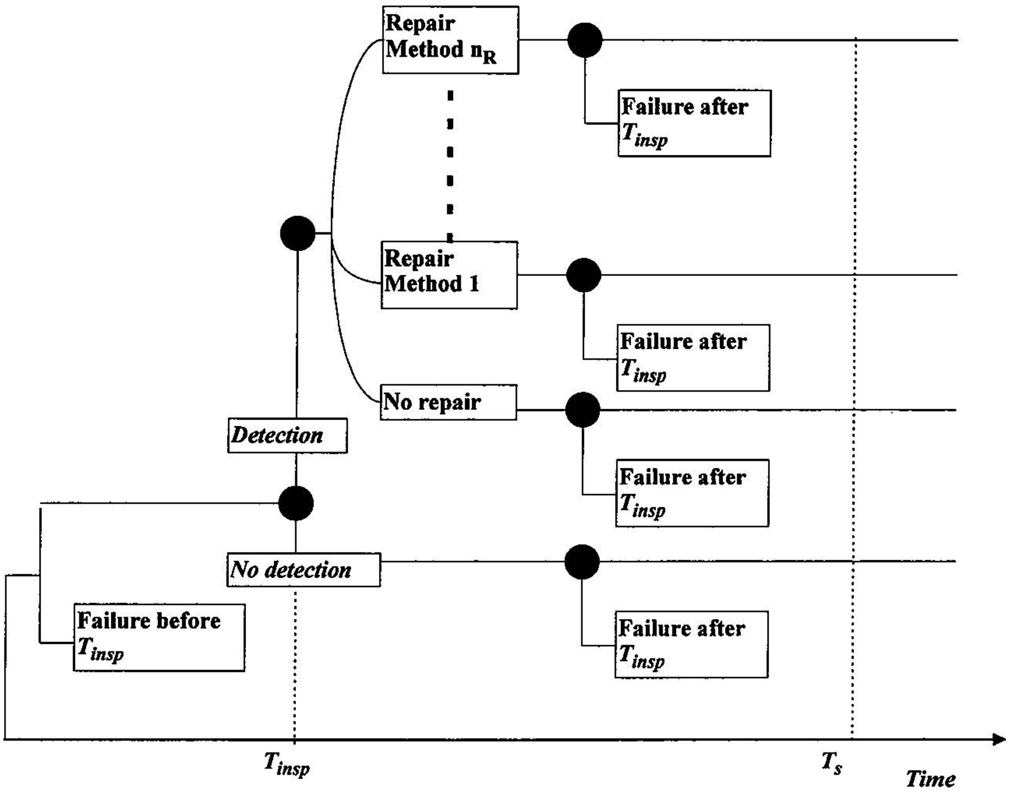

Fig.2 Events tree for a conditional maintenance model

3. Probabilities of failure and repair

Let us note $P_f(t)$ the probability of failure at time t. The reliability index β is defined by (First Order Reliability approach):

$$\beta(t) = -\Phi^{-1}\left(P_f(t)\right) \tag{11}$$

If we note $\Delta P_f(t_i, t)$ the probability of failure in the time interval $]t_i, t]$, then it follows:

- for $0 \leq t \leq t_1$:

$$P_f(t) = \hat{P}_f(t) = P\left(M(t) \leq 0\right) \tag{12}$$

- for $t_1 \leq t \leq t_2$:

$$\begin{aligned} P_f(t) &= P_f(t_1) + \Delta P_f(t_1, t) \\ &= P_f(t_1) + \Delta P_f^0(t_1, t) + \cdots + \Delta P_f^{n_R}(t_1, t) \\ &= P_f(t_1) \\ &\quad + P\left(M(t_1) > 0 \cap H^0 \leq 0 \cap M^0(t) \leq 0\right) \\ &\quad \vdots \\ &\quad + P\left(M(t_1) > 0 \cap H^{n_R} \leq 0 \cap M^{n_R}(t) \leq 0\right) \end{aligned} \tag{13}$$

and so on for each inspection time. The probability of repair N.r ($r \geq 2$) is determined by

$$P_{rep}(t_1) = P(M(t_1) > 0 \cap H_r \leq 0) \tag{14i}$$

$$\begin{aligned} P_{rep}(t_2) &= P_{rep}^{0,r}(t_2) + \cdots + P_{rep}^{n_R,r}(t_2) \\ &= P(M(t_1) > 0 \cap H^0 \leq 0 \cap M^0(t_2) \cap H^{0,r} \leq 0) \\ &\vdots \\ &+ P(M(t_1) > 0 \cap H^{n_R} \leq 0 \cap M^0(t_2) \cap H^{n_R,r} \leq 0) \end{aligned} \tag{14ii}$$

and so on, for each inspection time.

4. Optimisation of the next inspection time

The problem consists in the determination of an optimal inspection time t_1 which minimises the total expected cost. The inspection time t_1 must fulfil the condition $t_1 \leq T_s$, where T_s corresponds to the time with a reliability equal to the minimum reliability $\beta_{\min}$ which can be accepted. The expected cost models are therefore the following:

- Expected inspection cost:

$$C_I(t_1) = C_{ins}(1 - P_f(t_1)) \frac{1}{(1+\alpha)^{t_1}} \tag{15}$$

- Expected repair cost:

$$C_R(t_1) = \sum_{r=2}^{n_R} C_{rep}(r) P_{rep}^r(t_1) \frac{1}{(1+\alpha)^{t_1}} \tag{16}$$

- Expected failure costs:

$$C_F(t_1) = C_f(P_f(t_1) - P_f(t_0)) \frac{1}{(1+\alpha)^{t_1}} \tag{17i}$$

$$C_F(T_s) = C_f(P_f(T_s) - P_f(t_1)) \frac{1}{(1+\alpha)^{T_s}} \tag{17ii}$$

where C_{ins}, $C_{rep}(r)$, C_f are the expected inspection cost, the expected repair cost and the expected failure cost respectively, and α is the rate of interest.

The inspection time t_1 is therefore determined as the optimal solution of the minimisation problem

$$\min_{t_1} C_T(t_1) = \min_{t_1}(C_I(t_1) + C_R(t_1) + C_F(t_1) + C_F(T_s)) \tag{18}$$

The time t_0 can be any time after the putting in service of the welded joint. The time t in the models has nevertheless to be adjusted in order to take into account of this delay.

5. Optimisation of the next inspection time with observation

Inspections provide useful information for updating component reliability. In that case, the probabilities of repair and failure have to be modified by using a bayesian approach which replaces all the probabilities by conditional probabilities. If qualitative and quantitative inspection results are available, then the updated probability of failure can be expressed by :

- for $0 \le t \le t_1$:

$$P_f^{up}(t) = P\left(M(t) \le 0 \,/\, \bigcap_{i=1}^{p} \left(\tilde{H}_i^1 \le 0\right) \cap \bigcap_{i=1}^{q} \left(\tilde{H}_i^2 = 0\right) \right) \tag{19i}$$

where :

$\left(\tilde{H}_i^1\right)_{1 \le i \le p}$ and $\left(\tilde{H}_i^2\right)_{1 \le i \le q}$ are the qualitative and quantitative inspection results respectively.

- for $t_1 \le t \le t_2$:

$$\begin{aligned} P_f^{up}(t) &= P_f^{up}(t_1) + \Delta P_f^{up}(t_1,t) \\ &= P_f^{up}(t_1) + \Delta P_f^{up,0}(t_1,t) + \cdots + \Delta P_f^{up,n_R}(t_1,t) \\ &= P_f^{up}(t_1) \\ &+ P\left(M(t_1) > 0 \cap H^0 \le 0 \cap M^0(t) \le 0 \,/\, \bigcap_{i=1}^{p} \left(\tilde{H}_i^1 \le 0\right) \cap \bigcap_{i=1}^{p} \left(\tilde{H}_i^2 = 0\right) \right) \\ &\vdots \\ &+ P\left(M(t_1) > 0 \cap H^{n_R} \le 0 \cap M^{n_R}(t) \le 0 \,/\, \bigcap_{i=1}^{p} \left(\tilde{H}_i^1 \le 0\right) \cap \bigcap_{i=1}^{p} \left(\tilde{H}_i^2 = 0\right) \right) \end{aligned} \tag{19ii}$$

6. Optimisation problem

Here, the problem consists in the determination of an inspection interval Δt which induces a minimal maintenance expected cost during the conventional lifetime T of the component. For this purpose, the number of inspections n is first given, and then the total expected cost is evaluated. The procedure is performed for different number of inspections and the different expected costs are compared; the value n which provides the smallest cost gives to the optimal inspection period. Let us precise that some constraints have to be also fulfilled, as the optimisation problem beneath illustrates :

$$\min_{\Delta t} C_T(\Delta t) = \min_{\Delta t} \left[\sum_{i=1}^{n} \left[C_I(\Delta t) + C_R(\Delta t) + C_F(\Delta t) \right] + C_F\left(T_f t\right) \right] \tag{20i}$$

under constraints

$$\beta\left(T_f\right) \ge \beta_{\min};\ \Delta t_{\min} \le \Delta t \le \Delta t_{\max};\ 0 \le T_f - n\Delta t \le \Delta t_{\max} \tag{20ii}$$

Δt_{min}, Δt_{max} are minimal and maximal time intervals for inspections.

7. Example of a welded joint

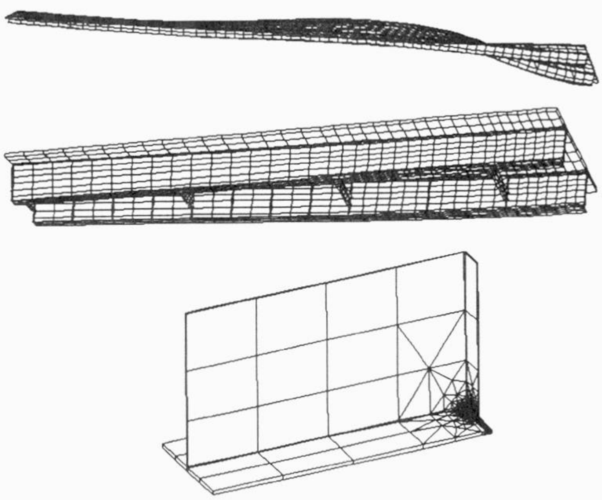

Fig 3 Finite elements modelling of the joint

A particular welded joint « bottom plate/stiffener » from a typical steel bridge has been here considered (Figure 1 and Figure 3). It is subject to 1.7 millions of stress variation cycles per year, with 11.0 MPa for mean. These values result from computations using the influence surfaces of the bridge and recorded heavy traffic data. The critical crack size is taken equal to the thickness of the bottom plate (crossing crack). The stress intensity geometry correction factor $Y(a, c)$ is the solution of Newman and Raju [5]. $Y(a, c)$ introduces the crack shape ratio a/c. This ratio is function of different parameters (local geometry, crack size a, stress intensity variation,...) which are difficult to model because of lack of information. For this reason, it is more suitable to use statistical distributions for describing the crack shape ratio according to the type of joint. For transversal welded joints, Yamada and al. [6] propose to choose lognormal distributions.

$$Y(a,c) = \frac{1}{\sqrt{1 + 1.464\left(\frac{a}{c}\right)^{1.65}}}\left[Y_1(a,c) + Y_2(a,c)\left(\frac{a}{b}\right)^2 + Y_3(a,c)\left(\frac{a}{b}\right)^4\right]$$

$$Y_1(a,c) = 1.13 - 0.09\frac{a}{c}; \quad Y_2(a,c) = \frac{0.89}{0.2 + \frac{a}{c}} - 0.54 \qquad (21)$$

$$Y_3(a,c) = 0.5 - \frac{1}{0.65 + \frac{a}{c}} + 14\left(1 - \frac{a}{c}\right)^{24}$$

The stress intensity concentration factor $M_k(a,b)$ is given by an exponential model:

$$M_k(a) = v\left(\frac{a}{b}\right)^w \qquad (22)$$

According to table 1 which provides the statistical properties of the different model variables, it is possible to evaluate the evolution of the reliability index related to the probability of failure as a function of the time t (Figure 4) given by :

$$\hat{P}_f = P\left(M(t) \le 0\right) \qquad (23)$$

Variable	Type	Mean	C.O.V.	Unit
a_0	Ln.	*0.0006*	*5%*	*m*
$a_c = b$	D.	*0.0175*	/	*m*
a_{rep}	Ln	*0.001*	*10%*	*m*
a_2	D	*0.003*	/	*m*
a_d	Ln.	*0.002*	*10%*	*m*
m	N.	*2.85*	*5%*	*adim*
C	Ln.	8.10^{-12}	*10%*	*adim*
v	Ln.	*0.77*	*2%*	*adim*
w	N.	*-0.24*	*6%*	*adim*
a/c	Ln.	*0.39*	*4%*	*adim*
v_0	D	*1700000*	/	*cycles/year*
ΔK_{th}	D	*0*	/	*MPa*
$E(\Delta S)$	D.	*11.0*	/	*MPa*
b	D.	*0.035*	/	*m*

***Table 1** Statistical characteristics*
($\rho(\ln C, m) = -0.9$ *and* $\rho(\ln v, w) = 0.99$)

Repair cost)	*4.2%*
Inspection cost	*0.2%*
Failure cost	*100%*
Conventional lifetime T_f	*100 years*
Rating of interest α	*4%*
Δt_{min}	*5 years*
Δt_{max}	*20 years*
β_{min}	*3.5*

***Table2** Optimisation characteristics*

The minimum reliability index β_{min} is obtained for the time $t \approx 22\ years$. This time will be used as the reference period T_s for the determination of the next inspection time.

The optimisation problem introduced in Section 4 is used for determining the first inspection time. No information is available, and the costs are all expressed in terms of percentages of the cost of failure [7]. The different expected costs can be calculated versus the next inspection time $t_1 \leq T_s = 22\ years$.

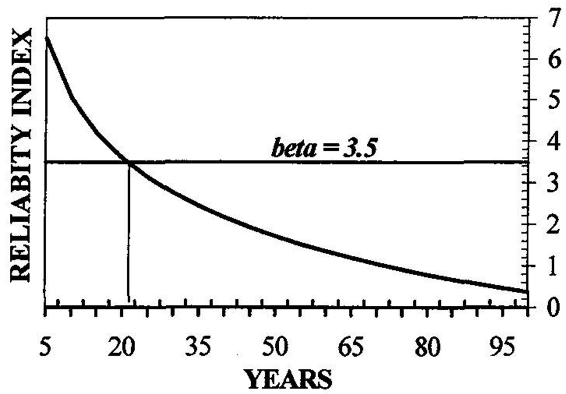

***Fig.4** Welded joint time-varying reliability*

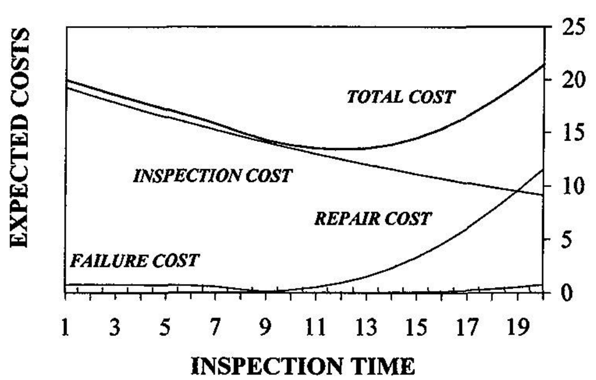

***Fig.5** Expected cost variation as a function of next inspection time*

Figure 5 illustrates the evolution of these costs according to the data of Table 2. The conditional maintenance strategy has three issues: no detection, detection with no repair and detection with repair by welding. The corresponding action intervals I_i are respectively $[0, a_d[$, $[a_d, a_2 = 3\ mm[$ and $[a_2, b[$. The computations of the probabilities of failure and repair are obtained through optimised recursive schemes requiring multidimensional integrals for the calculus of the probabilities in event series systems [7]. For this welded joint, the minimum total expected cost is obtained for an inspection time $t_1 = 12\ years$. The reliability index $\beta(20) = -\Phi^{-1}(P_f(20))$ is 4.6.

Figure 5 shows that the expected repair cost is non zero; consequently, a repair has to be expected at $t = 12$ *years.*

The optimal inspection interval is given by the optimisation problem of section 6. To solve this problem is very time consuming, and a lot of work has to be done for improving it. The optimal inspection interval is of *10 years*. The final reliability index is *4.0*.

8. Conclusions

The paper has presented a methodology for optimising the inspection programme for components in steel bridges. The concepts have been applied to welded joints with respect to fatigue failure and an example is given for highlighting the different theoretical aspects. The optimised inspection intervals are regular intervals, but the approach of Section 6 can be generalised to variable inspection intervals [7], but the corresponding computations are time consuming. It has to be noticed that the individual inspection intervals have to be combined, in a final stage, for providing a final inspection interval at the structure level. This can be performed by using qualitative combinations (expensive actions regrouping, available budget,...) as well as quantitative combinations of the individual inspection intervals [8]. Some additional effort is nevertheless still needed in order to assess the sensitivity of the results to change in the parameters involved in the cost optimisation procedure.

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