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Feedback and Sensitivity and their Measurement in Integrated Circuit Feedback Amplifiers

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1 Introduction

The concept of return-difference is fundamental to the study of performance of feedback amplifiers. The most common methods of measurement of return-difference involve opening the feedback loop and simulating network admittances. In the modern communications equipment the trend is definitely towards building amplifiers using integrated or thin-film circuit techniques and the opening of the feedback loop becomes extremely difficult or even impossible. For this reason the development of new test methods which do not require the modification of networks seems highly desirable. Furthermore, measurement methods without modifying the circuit are, in principle, experimentally superior, particularly for high frequency amplifiers (60 MHz and above) where measurements up to several GHz are made and the actual physical layout becomes critical in determining the network response.

A physical system can be characterized by a differential equation expressing a specific excitation-response relationship. A linear network is said to be asymptotically stable if its forcefree response tends to zero as time increases. This in turn means that the network determinant $\triangle$ (p), or characteristic polynomial, in the complex variable $p = \sigma + j\omega$ has only roots with negative real parts. Although a network can be represented on a mesh impedance-, nodal admittance-, or mixed-parameter matrix basis, in the following the nodal admittance characterization shall be used exclusively. By assumption, adding an admittance between two nodes, including a short circuit, is the only type of alteration allowed on the network, and this can most simply be described in terms of the nodal admittance matrix.

In practice $\triangle(p)$ can be evaluated only indirectly as the numerator or denominator of a measurable network function; e. g. the driving-point admittance of a network measured between the i^{th} and earth nodes is given by

$$Y_i = \frac{\triangle}{\triangle_{ii}}$$

where $\triangle_{ii}$ is the cofactor of y_{ii}, and it is the network determinant when the i^{th} node of the network is earthed. If the network is known to be stable with the i^{th} node earthed, i.e. $\triangle_{ii}$ has all its zeros in the left half-plane, then the original network is stable if the *Nyquist* plot of $Y_i(j\omega)$ does not encircle the origin.

When the network function is a transfer function from node j to node k, then

$$H(p) = \frac{\triangle_{jk}}{\triangle}$$

If the Nyquist plot of $H(j\omega)$ does not encircle the origin, then the network is stable if $\triangle_{jk}$ has no zeros in the right half-plane or, in other words, H is a 'minimum-phase' transfer function. This does not mean, however, that a transfer function has to be minimum phase (cf. Sect. 5). From a practical point of view the zeros in the right half-plane should be shifted as high out of the useful frequency band as possible to minimize their contribution to the in-band phase shift of the amplifier in order to avoid an undue reduction of the phase margin which in turn have to be compensated for by a reduction in the available feedback.

Another very useful function which is simply related to $\triangle(p)$ can be obtained when $\triangle(p)$ is explicitly given as a function of one of the network parameters g

$$\triangle = \triangle^\circ + g\triangle' \qquad (1)$$

where $\triangle^\circ$ and $\triangle'$ are in general combinations of determinants and cofactors each independent of g. Through dividing both sides of eqn. 1 by $\triangle^\circ$ the return-difference function as defined by *Bode*[1] results

$$F_g(p) = \frac{\triangle}{\triangle^\circ} = 1 + \frac{g\triangle'}{\triangle^\circ} \qquad (2)$$

Again, as far as the stability of the network is concerned, $F_g(j\omega)$ provides the same information about the network as $\triangle(j\omega)$ itself does, given that the network is stable when a specific element vanishes. The variable term in eqn. 2 will be recognized as the return-ratio T_g which is the negative of the loop-gain. For $T_g(j\omega)$ the Nyquist stability criterion requires that it should not enclose the critical point (–1,0). The methods of measuring loop-gain by opening the feedback loop and the problems associated with it have been described by *Hakim*[2] and *Hoskins*[3].

2 Feedback and sensitivity

The usual concept of feedback includes two distinct ideas. The first is that of 'transmission around a loop' or return of a signal. In terms of the fundamental signal flow graph for feedback systems this looks deceptively simple. All that is required is to break the loop at one point, apply a signal, then calculate the ratio of the returned signal and the input to obtain the loop-gain. In actual physical systems when breaking the loop it has to be terminated so that from the point of view of the controlled source nothing has changed and, at the same time, the controlled source does not influence its own controlling signal. The assumption that for an arbitrary network the loop can be terminated in such a way using a two-pole can lead to unjustified generalizations[4]. But, as it will be seen later, the laxity at this point is far from being the most important source of error in the uses of feedback.

It can be argued that feedback loops are present in the topological characterization of networks simply because of the form of equations one has chosen to write[3,5]. To illustrate this point[6] it will suffice to recall that the equations of state of a passive RLC-network have precisely the same form as those representing the fundamental signal flow graph for feedback analysis, yet one seldom looks for feedback loops in RLC-networks.

The second idea associated with feedback is that of reduction of the effects of component variation on some network function. This idea of a reduction in the effects of component variation will be referred to as 'sensitivity'. In most networks the concept of 'transmission around a loop' or feedback and that of 'sensitivity' are simply related and can be used interchangeably.

The sensitivity function as used in modern literature is the inverse of that first proposed by Bode[1]

$$S_g{}^H(p) = \frac{d[\ln H(p, g)]}{d(\ln g)} = \frac{dH(p, g)/H(p, g)}{dg/g} \tag{3}$$

$S_g{}^H$ is called the sensitivity function of the network function H as a function of small changes in the parameter g, and it is the ratio of the % change of the value of the function and that of the parameter g.

The transfer-impedance Z_{ik} for an amplifier represented schematically in *Figure 1* is numerically the same as the current-transfer ratio H for a load admittance G_1 normalized to unity. Let $g = y_{rs}$ for a unilateral element and y_{rr} or y_{ss} for a bilateral one, then

$$H(p, y_{rs}) = \frac{\triangle_{jk}}{\triangle} = \frac{\triangle^\circ{}_{jk} + y_{rs}\triangle_{jkrs}}{\triangle^\circ + y_{rs}\triangle_{rs}} \tag{4}$$

where $\triangle$ is the determinant of the nodal admittance matrix of the amplifier including G_s and G_1. To be able to expand $\triangle$ as shown in eqn. 4 one has to assume that y_{rs} appears in only one position in the matrix, i. e. when g is a bilateral element it is connected to the reference node and when it is the forward transfer admittance of a transistor, there is no local feedback to that stage.

This assumption is made in order to make the algebra simpler and the results readily interpretable, but it in no way restricts the validity of the equation. The essential feature of eqn. 4 is that a network function can be expressed as a bilinear function of a single variable g regardless of where this element is connected in the network[1]. Through formal manipulation to eqn. 4 one gets

$$\begin{aligned} H(p,y_{rs}) &= \frac{\triangle^\circ{}_{jk}/\triangle_{rs} + y_{rs}\triangle_{jkrs}/\triangle_{rs}}{\triangle^\circ/\triangle_{rs} + y_{rs}} \\ &= \frac{(\triangle^\circ{}_{jk}/\triangle^\circ)(\triangle^\circ/\triangle_{rs}) + y_{rs}\triangle_{jkrs}/\triangle_{rs}}{\triangle^\circ/\triangle_{rs} + y_{rs}} \end{aligned} \tag{5}$$

where $\triangle^\circ$ is $\triangle$ with y_{rs} removed (open-circuited) and $\triangle_{rs}$ when y_{rs} is shorted out. Thus $\triangle^\circ/\triangle_{rs}$ is the *Thevenin* admittance Y facing element y_{rs} in the network. $\triangle_{jkrs}$ is $\triangle$ with its rows j,r and columns k,s eliminated. By virtue of eqn. 4 $\triangle^\circ{}_{jk}/\triangle^\circ = H(p,0)$ is the value of the transfer function when $y_{rs} = 0$, and $\triangle_{jkrs}/\triangle_{rs} = H(p,\infty)$ is the value of H when $y_{rs} = \infty$. Thus eqn. 5 can be written in the following form

$$H(p,y_{rs}) = \frac{Y(p)H(p,0) + y_{rs}H(p,\infty)}{Y(p) + y_{rs}} \tag{6}$$

The sensitivity function of the transfer function $H(p,y_{rs})$ can be obtained by performing the operations indicated in eqn. 3

$$S^H{}_{y_{rs}} = \frac{Y \cdot y_{rs}[H(p,\infty) - H(p,0)]}{(Y + y_{rs})[y_{rs}H(p,\infty) + Y \cdot H(p,0)]} \tag{7}$$

If in eqn. 7 $H(p,0) = 0$, then

$$S^H{}_{y_{rs}} = \frac{Y}{Y + y_{rs}} = \frac{\triangle^\circ/\triangle_{rs}}{\triangle^\circ/\triangle_{rs} + y_{rs}} = \frac{\triangle^\circ}{\triangle} = \frac{1}{F_{y_{rs}}(p)} \tag{8}$$

Eqn. 8 expresses the important fact that the sensitivity of a network function with respect to an element of a network is equal to the inverse of the return-difference to that element if its vanishing ($y_{rs} = 0$) results in zero value for the given function. Also the effects of variation of a network element on a network function are reduced proportionally to the return-difference to that element. The same statement can be made of the reduction of distortion signal generated by an element, the truth of which readily follows from the *Compensation Theorem* for electric networks. Furthermore, eqn. 8 shows that the admittance measured between any two nodes of a network is the negative of the admittance which, when connected in parallel at the same nodes, just causes the network to oscillate.

If in eqn. 8 one sets $Y = k_1 y_{rs}$, where k_1 is a real non-zero constant, i. e. the Thevenin admittance Y has the same poles and zeros as y_{rs}, then

$$S^H{}_{y_{rs}} = \frac{k_1 y_{rs}}{k_1 y_{rs} + y_{rs}} = \frac{k_1}{k_1 + 1} \tag{9}$$

the sensitivity function is frequency-independent and 'perfect gain control'[7] is obtained. If, in general, $H(p,\infty) = k_2 H(p,0)$, where $k_2 \neq 0$ or 1 and $H(p,0) \neq 0$, then eqn. 7 yields

$$S^H{}_{y_{rs}} = \frac{k_1}{k_1 + 1} \cdot \frac{k_2 - 1}{k_2 + 1} \tag{10}$$

and again 'perfect gain control' obtains. A case of obvious interest occurs when control can be achieved by varying a single element, e. g. a resistor such as in resistance networks or in RLC-equalizers of a constant resistance type where Y is purely resistive at some nodes. When Y is a complex admittance function, the results of eqn. 9 can be applied to justify the wellknown procedure to obtain arbitrary termination ratios for physically symmetrical networks as shown in *Figure 2.* By assumption $Y_a = k_1 y_{rs}$, thus y_{rs} can be realized as a two-pole having exactly the same structure as Y_a with its admittance properly scaled. Therefore y_{rs} and $\frac{1}{2}Y_a$ can be combined element by element in parallel

$$rY_a = \tfrac{1}{2}Y_a + \frac{Y_a}{k_1}, \text{ where } r = (k_1 + 2)/2k_1$$

It may be noted here that, while the roots of the transfer function remain invariant to this transformation, those of the input and output admittances do not.

From a measurement point of view eqn. 7 is not very useful in this form since, if y_{rs} is the transfer ratio of a dependent source, $H(p,\infty)$ is not physically realizable. From eqns. 3 and 4

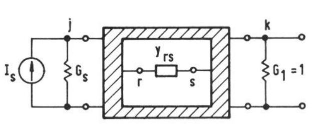

Fig. 1
Schematical representation of an element y_{rs}

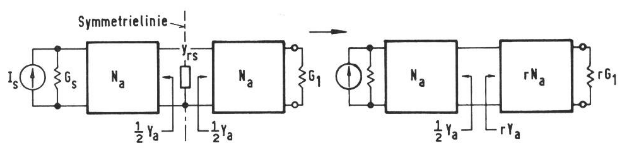

Fig. 2
Illustration of transfer function scaling
Symmetrielinie – Symmetry line

$$S^H_{y_{rs}} = \frac{y_{rs}\Delta_{jk}}{\Delta} \cdot \frac{\Delta\Delta_{jkrs} - \Delta_{jk}\Delta_{rs}}{\Delta^2}$$

By making use of the expansion of the determinant in eqn. 4, for the above expression obtains

$$S^H_{y_{rs}} = \frac{\Delta^\circ}{\Delta} - \frac{\Delta^\circ_{jk}}{\Delta_{jk}} = \frac{1}{F(p)} - \frac{1}{F_o(p)} \tag{11}$$

$F_0 = \Delta_{jk}/\Delta^\circ_{jk}$ can be identified with the so-called *null-return-difference* and it can be shown to have a distinct physical interpretation in every concrete case. Thus when $H(p,y_{rs})$ is a current gain function, then formal manipulation yields

$$F_o = \frac{\Delta^\circ_{jk}}{\Delta_{jk}} \cdot \frac{\Delta}{\Delta^\circ} \cdot \frac{\Delta^\circ}{\Delta} = \frac{H(p,y_{rs})}{H(p,0)} \cdot \frac{1}{F(p)}$$

By using this expression for F_0 in eqn. 11 one obtains

$$S^H_{y_{rs}} = \frac{1}{F(p)}\left[1 - \frac{H(p,0)}{H(p,y_{rs})}\right] \tag{12a}$$

The above expression for the sensitivity of the function $H(p,y_{rs})$ is a very useful one in that it not only says that sensitivity is equal to the inverse of the return-difference for any element y_{rs} such that $H(p,y_{rs} = 0) = 0$, but it is also possible to obtain the absolute sensitivity by measuring two network functions. In other cases it provides an estimate of the error in assuming that the return-difference is an accurate measure of sensitivity. If, e. g. the ratio $H(p,0)/H(p,y_{rs}) = -40$ dB, this error is at most 1%, which is entirely negligible. When $H(p,0)$ and $H(p,y_{rs})$ are nearly equal in magnitude and phase, the return-difference gives a very pessimistic estimate of sensitivity. This is usually the case when in a single-loop amplifier local feedback is added to a stage. While the feedback to that stage remains about the same, the return-difference measured in the main loop decreases. But as long as the return-difference in the main loop remains fairly large (e. g. 20 dB) the change in the transfer function in mid-band will be hardly noticeable.

On the other hand, when $H(p,0) \gg H(p,y_{rs})$, the return-difference becomes a very optimistic estimate. This situation arises when, e. g. y_{rs} represents the feedback resistor y_f of a single-loop amplifier. When $y_f = 0$ (opening the feedback loop) the output increases. To demonstrate this, first the open-loop gain or 'gain before feedback'' is defined. From eqn. 5

$$H(p,y_{rs}) - H(p,0) = \frac{y_{rs}(\Delta^\circ\Delta_{jkrs} - \Delta_{jk}\Delta_{rs})}{\Delta^\circ(\Delta^\circ + y_{rs}\Delta_{rs})} \tag{12b}$$

With the aid of *Jacobi*'s theorem: $\Delta_{jkrs} = \Delta_{jk}\Delta_{rs} - \Delta_{js}\Delta_{rk}$ eqn. 12b, after multiplying by $\Delta^\circ/\Delta^\circ$, can be written as

$$H(p,y_{rs}) - H(p,0) = -\frac{y_{rs}\Delta_{js}\Delta_{rk}}{\Delta^\circ(\Delta^\circ + y_{rs}\Delta_{rs})} \cdot \frac{\Delta^\circ}{\Delta^\circ} = \frac{1}{F} \cdot H'(p,y_{rs}) \tag{12c}$$

where the expression associated with $H'(p,y_{rs})$ is called by Bode the 'gain before feedback' or 'fractionated gain'. Physically this means that, when measuring the open-loop gain, the loading effects of the feedback circuitry are taken into account when the broken loop is properly terminated. Eqn. 12c also indicates that gain reduction is applied only to the surplus of the total output over direct transmission. Since the return-difference to y_f is the same as to any of the transistors, from eqn. 12a the wellknown result is obtained

$$S^H_{y_f} = \frac{1}{F}\left(1 - \frac{H'}{H}\right) = \frac{1-F}{F} = -\frac{T}{1+T} \tag{12d}$$

3 Non-uniqueness of loop-gain and sensitivity

So far changes in a network function due to a single parameter only have been considered. In reality the values of all the components comprising the network are subject to normal manufacturing tolerances. The corresponding normalized change in the network function $H(p,g_i)$ can be estimated from the relation

$$\frac{\Delta H}{H} \approx \frac{dH}{H} = \sum_{i=1}^{N} S^H_{g_i} \frac{dg_i}{g_i} \tag{13}$$

where $S^H_{g_i} = \frac{d\ln H}{dg_i}$ and N is the number of components in the network. The quantity $S^H_{g_i}\frac{dg_i}{g_i}$ is called the *variation* $V^H_{g_i}$ of the parameter g_i. While with the aid of a digital computer the sensitivity analysis of a network presents no particular difficulties, from a measurement point of view it is necessary to reduce the list of variables to a manageable size; namely to those parameters whose variations dominate the sum in eqn. 13. In active networks they are usually the variations of the transistor gain parameters for which dg_i/g_i is likely to be very large, although their sensitivity may be in the same range as that of the passive elements. When a network contains a variable parameter g_i, the poles and zeros of a network function are also variables in g_i. Since the poles and zeros of a network determine its transient as well as its steady state behaviour, the sensitivities of the poles and zeros give a quantitative measure of the extent to which network functions may vary with g_i. A network can be uniquely described by its poles p_k and zeros z_k and a scale factor K

$$H(p,g_i) = K\frac{\prod_k (p + z_k)}{\prod_k (p + p_k)} \tag{14}$$

By applying eqn. 3 to eqn. 14, one obtains:

$$S^H_{g_i} = S^K_{g_i} + \sum_k \frac{S_{g_i}{}^{p_k}}{p + p_k} - \sum_k \frac{S_{g_i}{}^{z_k}}{p + z_k} \tag{15}$$

where $S_{g_i}{}^{p_k}$ and $S_{g_i}{}^{z_k}$ are the pole and zero sensitivities, respectively defined by

$$S_{g_i}{}^{p_k} = g_i \frac{dp_k}{dg_i} \text{ and } S_{g_i}{}^{z_k} = g_i \frac{dz_k}{dg_i}$$

These equations will be recalled later on.

The transistor, being a non-reciprocal device, can be completely described by four sets of measurements of its driving-point and transfer characteristic. Since sensitivity and return-difference are specified with respect to a network parameter having a one to one relationship to a piece of hardware or constraint (R,L,C, control element of a dependent source, etc.) in the network, it makes little sense to speak about the sensitivity or return-difference to a transistor until a transistor model has been chosen. But a transistor can be represented in a variety of ways each consisting of a number of bilateral elements and one or two dependent sources: In each case the sensitivity will be different. Furthermore, sensitivity depends to a large extent on the type of network function one wishes to investigate, thus no single 'best' sensitivity model of the transistor can exist. In the final analysis, the choice of a transistor model is dictated by the necessity of being able to define a return-difference which is a measurable network function so that it can be of some use in an actual stability investigation. Sometimes this choice is simply a matter of convenience, e. g. when the sensitivity of a transfer function is desired, the controlled source to which the return-difference is sought is such that the forward transmission through the transistor is interrupted when that source is put to zero. In this way the return-difference measured in a single-loop amplifier becomes also the measure of sensitivity. Sometimes the transistor as a whole is taken as a single variable by considering all the transistor parameters as a 'sensitivity group'[8], since a change in one of the parameters usually affects all the others.

4 Single-loop amplifiers

A single-loop amplifier, illustrated schematically in *Figure 3,* consists essentially of a forward gain- (μ-circuit) and a feedback path (β-circuit) with combining circuits at both the input and output of the amplifier. For the transistors, represented by their y-parameters, y_{12} is assumed to be zero. In this way there is no transmission in the reverse direction in the μ-circuit. It is also reasonable to suppose that under normal operating conditions the signal transmitted through the β-circuit to the load is a negligible portion of the total. When a bridge-type combining circuit very common in amplifiers is used at either the input or the output of the amplifier, the above assumption is fully justified. It is further assumed that the amplifier performance can be adequately described in terms of the control parameter y_{21} of the dependent source of each transistor responsible for the forward transmission.

When the return-difference is evaluated for more than one dependent source simultaneously, it can be described by the *Tasny-Tchiassny*[9] return-difference matrix [F]. For the amplifier in Figure 3

$$[F] = \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \qquad (16)$$

where F_{ij} expresses the dependence of the controlling variable of the i^{th} controlled source on the controlling variable of the j^{th} controlled source. The components of the 1^{st} column of the matrix may be evaluated by letting the control parameters of the dependent sources of TR2 and TR3 assume their reference value of $y_{21} = 0$ and replacing the dependent source of TR1 by an independent source of unit strength. The returned controlling signals developed for the three dependent sources are the elements of the first column, respectively. By repeating the above procedure of the remaining controlled sources in turn, the complete matrix is obtained. After evaluating the return-difference matrix for the amplifier in Figure 3 it is readily seen that

$$F_{11} = F_{22} = F_{33} = 1 \text{ and } F_{31} = F_{12} = F_{23} = 0.$$

The last set of equations means that there is no reverse transmission in the μ-circuit and in the forward direction only to the controlling-voltage node of the adjacent transistor. When all the control parameters are restored to their normal value, the return-difference F_i to the i^{th} transistor is obtained from eqn. 16 as

$$F_i = \frac{\det[F]}{\det[F]_{ii}} \qquad i = 1, 2, 3 \qquad (17)$$

where $\det[F]_{ii}$ is obtained by deleting the i^{th} row and column of [F]. For a three stage amplifier

$$F_i = 1 + F_{21}F_{32}F_{13}, \quad i = 1,2,3 \qquad (18)$$

i. e. to all three control parameters the return-difference is the same. Eqn. 18 is the theoretical basis of measuring the return-ratio to a transistor gain control parameter in a single-loop amplifier by breaking the feedback loop at *any* convenient point. Then, after terminating the loop in an impedance normally seen at that point looking in the direction of transmission, a loop transmission measurement yields the return-ratio. Since single-loop amplifiers are known to be stable when the feedback loop is opened, the amplifier remains stable when feedback is restored if the Nyquist plot of $F_i(j\omega)$ does not encircle the origin of the F-plane.

In practice it is also of great interest whether an amplifier remains stable when exposed to the inevitable component variations. This concern is usually expressed in terms of safety margins against oscillation which, for the loop gain of a single-loop amplifier, are usually given as 30° phase margin and 10 dB gain margin. This is illustrated in *Figure 4.*

When local feedback exists for one of the stages of the amplifier of Figure 3, it can still be regarded as a single-loop amplifier with minor modifications. For example, if there is shunt feedback across TR2, the return-difference matrix becomes

$$[F] = \begin{pmatrix} 1 & 0 & F_{13} \\ F_{21} & F_{22} & 0 \\ F_{31} & F_{32} & 1 \end{pmatrix}$$

from which, by virtue of eqn. 17, one obtains:

$$F_1 = F_3 = \frac{\det[F]}{\det[F]_{11}} = \frac{\det[F]}{\det[F]_{33}} = \frac{F_{22} + F_{21}F_{32}F_{13} - F_{31}F_{22}F_{13}}{F_{22}} \qquad (19)$$

which reduces to unity when either TR1 or TR3 is inactive. Thus from the point of view of these stages the amplifier is still single-loop and the loop-gain can be measured in the usual way. The return-difference to these stages, however, is reduced by approximately the amount of the local feedback for the second stage TR2. For TR2

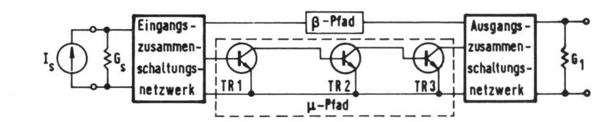

Fig. 3
Simplified schematic of a single-loop amplifier
Eingang-/Ausgangzusammenschaltungsnetzwerk – Input/Output interconnecting network

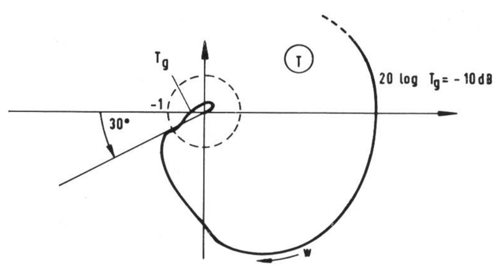

Fig. 4
Nyquist locus of the loop-gain of a typical amplifier

$$F_2 = \frac{\det[F]}{1-F_{31}F_{13}}$$

which is equal to F_{22} when TR1 and/or TR3 is inactivated. The total return-difference for this stage is about the same as it would be without local feedback. However, the sensitivity of the transfer function is not the inverse of the return-difference of this stage, since the output is not reduced to zero when TR2 is inactivated. In this case the sensitivity may be calculated from the results of two measurements using eqn. 12. For a single-loop amplifier a very interesting relation can be derived for the sensitivity of the transfer function to transistor gain parameters. From eqns. 3 and 4, after some algebraic manipulations, one obtains

$$S^H_{y_{rs}} = \frac{H(p,\infty) - H(p,y_{rs})}{y_{rs\infty} - y_{rs}} \cdot \frac{y_{rs}}{H(p,y_{rs})} \qquad (20)$$

where $y_{rs\infty} = -\triangle^\circ/\triangle_{rs}$, by eqn. 4, is the value of y_{rs} corresponding to an infinite value of the function H. In terms of the loop-gain of Figure 4 this happens when, as the loop gain increases with y_{rs}, the curve will just go through the (–1,0) point and the amplifier is just beginning to oscillate. For an amplifier with an overall feedback of 30 dB, the error is very small in assuming that the flat gain is equal to $H(p,\infty)$. Suppose that it is required that $|S_y^H| \leq 1.6$. This means that the loop-gain curve in Figure 4 must not enter the circle of radius 0.625 centered at (–1,0). This condition is roughly equivalent to the 30° phase margin and 10 dB gain margin mentioned earlier. If the overshoot, which occurs approximately at the frequency where the loop-gain curve just touches the forbidden disc, is 3 dB with respect to the flat gain then from eqn. 20, $y_{rs} = 1.19\, y_{rs}$. This means that for a three stage amplifier an increase of 19% in the gain of one of the transistors, or a 6% simultaneous increase in that of all three, will cause oscillation. Fortunately this allowable % variation increases rapidly with internal feedback, which is always present in a transistor. Internal feedback can be further increased by the addition of external local feedback to obtain satisfactory performance even for gain variations of 100 – 200% common in practice.

These results obtained for feedback are at variance with those of a typical signal flow diagram analysis where it is routinely shown that a single feedback loop around all the stages results in a smaller sensitivity than purely local feedback or the combination of the two. This type of analysis, however, does not take into account the band-limited nature of the transistor.

41 Measurement of return-difference based on driving-point- and transfer-admittance measurements

These methods are based on *Blackman*'s classical results[10], which can be very elegantly derived[1], e. g. for the driving-point admittance by formal manipulation of the equation for the admittance between node i and earth:

$$Y_i = \triangle/\triangle_{ii} = \triangle^\circ/\triangle^\circ_{ii} \cdot \triangle/\triangle^\circ \cdot \triangle^\circ_{ii}/\triangle_{ii} = Y_o \cdot F \cdot 1/F_o \qquad (21)$$

where Y_0 is the admittance at node i measured when the parameter, to which the return-difference is sought, is set equal to zero. F_0 is the return-difference for the same element when node i is shorted to ground. If the admittance is measured at a node such that its grounding reduces that particular element to zero, then $F_o = 1$. In terms of the network determinant $\triangle$ this means that, e. g. a unilateral control element is located in the i^th^ row or column, therefore $\triangle_{ii} = \triangle^\circ_{ii}$. Eqn. 21 can be used to measure return-difference by measuring Y and Y_0. Y is measured for normal operating conditions and Y_0 when a particular parameter is set to zero. Y_0 can be measured by opening the loop between TR1 and TR2 of the amplifier in Figure 3, terminating TR1 in y_{11} of TR2, and shorting the base of TR2 to ground[2]. The admittance measured under these conditions at the collector of TR1 is Y_0. Or, to avoid simulating y_{11}, Y_0 can be obtained under the same conditions as the sum of two measurements made into the network ports thus created with the other port shorted, alternately[11]. However, it is not necessary to open the feedback loop to measure Y_0. In Section 4 it was shown that the loop-gain for any transistor can be set to zero by interrupting the feedback loop at any convenient point. Furthermore, if the admittance is measured, e. g. at the node between TR1 and TR2, and the loop is interrupted between TR2 and TR3 then, by the definition of a single-loop amplifier, there is no way to experimentally determine at the point of measurement whether the path is broken or shorted to earth. In practice, to minimize the effects of local feedback on TR2, the path is shorted to earth at a point far removed from the point of measurement, e. g. at the collector of TR3. The short is made with a capacitor to keep dc bias intact.

Return-difference can be measured under similar conditions without opening the feedback loop by making two transmission measurements with a vector voltmeter

$$F = \triangle/\triangle^\circ = \triangle/\triangle_{ki} \cdot \triangle_{ki}/\triangle^\circ \qquad (22)$$

where k is, e. g. the input node to the amplifier and node i is at the base of a transistor; therefore $\triangle_{ki} = \triangle^\circ_{ki}$. Thus $\triangle^\circ_{ki}/\triangle^\circ$ is the transmission from node k to node i when the transistor is inactive (loop is shorted out at a remote point) and $\triangle_{ki}/\triangle$ is that under normal operating conditions.

Eqn. 21 can be very effectively used also to evaluate the return-difference for a transistor stage in the presence of local feedback by making two measurements. Supposing there is series feedback (resistor in the emitter circuit) on TR1, then

$$Y_1 = Y_{01}F_1 \qquad (23)$$

where Y_1 and Y_{01} are measured at, e. g. the base of TR2 when the emitter resistor is shorted out and when the main loop is also shorted to ground at the collector of TR3. When measuring Y_1, however, the amplifier may oscillate when the emitter resistor is shorted out. In any case one may continue by restoring the emitter feedback resistor and measure admittances at the same point. For this condition

$$Y_2 = Y_{02}F_2. \qquad (24)$$

From eqns. 23 and 24 the amount of the return-difference $F_{loc.}$ due to local feedback is

$$F_{loc.} = F_1/F_2 \qquad (25)$$

This measurement is further discussed later on.

(To be continued)