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Feedback and Sensitivity and their Measurement in Integrated Circuit Feedback Amplifiers[1]

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If the removal of the local feedback results in oscillation, which is often the case, then Y_1 cannot be measured. In this case a resistor of known value may be connected across the terminals of measurement such that the oscillation ceases. The calculation of F and F_0 can proceed as before after subtracting the known value of the resistor from the measured values of Y_1 and Y_{01}. The purpose of adding a resistor is to shift the Nyquist admittance locus, which for an oscillator encircles the origin, to the right till it no longer does so. Even if the amplifier is stable, the admittance can still have negative real part in some frequency intervals and direct bridge measurement is not possible without making Re $[Y(j\omega)] \geq 0$ for all frequencies.

Since admittances measured by two different values of a parameter determine the return-difference, adding admittance between those same nodes may serve during the manufacturing process of an integrated circuit amplifier to shape the loop-gain so as to meet the safety margins against oscillation. For this purpose let

$$Y = Y_0(1+T) = Y_0 + Y_0 T \qquad (26)$$

at a suitable node of an amplifier. When an admittance Y_N is added to this node, eqn. 26 can be written

$$Y + Y_N = (Y_0 + Y_N)(1+T') \qquad (27)$$

From eqns. 26 and 27

$$T' = Y_0 T/(Y_0 + Y_N) \qquad (28)$$

If T' is the desired loop-gain, then a network Y_N may be synthesized to realize it if rational function approximations to T and T' are obtained first. From eqn. 28

$$Y_N = Y_0 \left(\frac{T}{T'} - 1\right) \qquad (29)$$

If the amplifier is not stable to begin with, then the procedure indicated following eqn. 25 may be implemented. When loop-gain correction is attempted at a node such that its grounding does not reduce the loop-gain to zero, then eqns. 26 to 29 have to be modified to the general form of Blackman's equation.

5 Multiple-loop feedback systems

An amplifier with one overall feedback loop but with local feedback on at least one of the transistors is, strictly speaking, no longer a single-loop amplifier. In this sense all physical amplifiers are multiloop. Local feedback is, however, a special case. In Section 41 feedback to a single transistor gain parameter was discussed, but to obtain it both the transistor gain parameter and the series feedback resistor had to be returned to their zero reference state. The ambiguity in this case is only apparent, and for the following reason: Instead of removing both parameters to interrupt transmission, the same result could have been achieved by assigning the transistor gain parameter a reference value other than zero such that the gain of the overall loop becomes zero. For the case of a general local feedback this reference value y_{210} of the gain parameter y_{21} of the transistor as a function of the feedback circuit elements can be easily determined[1].

$$y_{210} = \frac{Y_3Y_4 + Y_1Y_4 + Y_1Y_3 + Y_2Y_4}{Y_2 - Y_4} \qquad (30)$$

where Y_1 to Y_4 are the admittances of a transistor stage with both series and parallel local feedback as shown in *Figure 5.*

When only shunt feedback exists

$$Y_{210} = Y_4$$

and for series feedback only

$$Y_{210} = Y_1Y_3/Y_2$$

In either case varying Y_4 or Y_2 is equivalent to setting a different reference value for the gain parameter. By eliminating local feedback zero is established as the reference value for y_{21}.

In general, it can be shown that ratios of return-differences are constant[1]. Thus, e. g. if a network contains two elements k_1, k_2 of interest, then for the ratio of the return-differences one obtains:

$$\frac{F_1}{F_2} = \left[\frac{\triangle(k_1,k_2)}{\triangle(0,k_2)} \cdot \frac{\triangle(k_1,0)}{\triangle(k_1,k_2)}\right] \cdot \frac{\triangle(0,0)}{\triangle(0,0)} = \frac{\triangle(0,0)}{\triangle(0,k_2)} \cdot \frac{\triangle(k_1,0)}{\triangle(0,0)}$$

Although, in general, $F_1 = \triangle(k_1,k_2)/\triangle(0,k_2) \neq \triangle(k_1,0)/\triangle(0,0)$, and similarly for F_2, nonetheless their ratio is invariant regardless of the values the other parameters may have when the return-difference to a particular element is evaluated. This result is of great utility, e. g. in Section 4 1, when local feedback is calculated as the ratio of two return-difference measurements, but in one case there is a risk of oscillation. This possibility can be avoided in this case by measuring the return-difference first with the main loop disabled and secondly with both the main loop and local feedback eliminated. In calculating the network functions the algebra can often be simplified by setting most of the gain parameters to zero or some other convenient reference value[5].

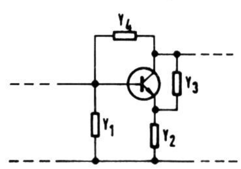

Fig. 5
An amplifier stage with both series and shunt feedback

[1] Part 1 appeared in No 3/1976, pp 109...113

Finding the reference value of a parameter in a single-loop amplifier is analogous to establishing a bridge-balance to decouple the input from the output. In a general multiple-loop amplifier, where feedback paths can exist from the output of any stage to the input of any stage, this balance condition is far from obvious. Nyquist's criterion (or *Argument Principle* for analytic functions) can, of course, still be applied to the multiple-loop case, too, to determine whether the amplifier is stable or not. To begin with, all transistors are inactivated to make sure that the amplifier is stable. Then the transistors are restored one by one to their normal operating condition and at each step the return-difference is measured and plotted. For, e. g. the j^{th} tranistor restored

$$F_j = \frac{\triangle(y_1, y_2, ..y_j, 0, 0, ..0)}{\triangle(y_1, y_2, .y_{j-1}, 0, 0, .0)} \qquad (31)$$

and the Nyquist diagram of F_j will encircle the origin clockwise $(z_j - p_j)$ times, where z_j and p_j are respectively the number of zeros and poles of F_j in the right half-plane. For all the n plots

$$\sum_{j=0}^{n} (z_j - p_j) = N_n$$

where N_n is the net encirclement of the origin. Since the numerator of F_j is the same as the denominator of F_{j+1}, for stability we must have $N_n = 0$ when all the transistors are restored to their normal value.

There are, however, n! different ways in which the transistors can be restored to their normal state. Although the final index of stability or instability must be independent of the order in which the transistors are restored, the return-difference diagram for any individual transistor may be vastly affected by the members of the set of transistors already activated when that particular transistor is restored. Thus the stability margins for return-differences mentioned earlier become meaningless in the multiple-loop case. Even though the individual return-differences are not uniquely obtainable for a multiple-loop amplifier, the determinant of the return-difference matrix as defined in Section 4 is unique. Blackman's equation, eqn. 21, can be generalized[5] to read

$$Y = Y_o \frac{\det[F]}{\det[F_o]} \qquad (32)$$

where $[F_0]$ is the return-difference matrix evaluated from the network when the terminal pair where Y is measured is short-circuited. While there is no theoretical difficulty in evaluating the return-difference matrix, in actual physical multiple-loop networks it is not possible to break a loop without affecting some of the others. Even for single loop amplifiers the open-loop gain has to be defined in a certain way to avoid the difficulties that can arise from network interconnections[12]. Thus, in general, it is difficult to establish a relationship between the return-differences as defined in eqns. 17, 31 and 32.

But eqn. 12c may be generalized for multiple-loop systems to read

$$\det[F] = \frac{\prod_i (p + a_i)}{\prod_j (p + b_j)} \qquad (33)$$

where a_j and b_j denote the set of closed-loop and open-loop characteristic frequencies, respectively. The denominator of eqn. 33 is that of the 'gain before feedback' defined earlier.

Eqn. 33 can be factorized in terms of the eigenvalues or, more precisely, the characteristic transfer functions f_j of det [F]

$$\det[F] = \prod_{j=1}^{n} f_j \qquad (34)$$

where n is the number of control parameters. Eqn. 17 gives the same results for the return-differences, i. e. $f_i = F_i$ if the matrix [F] is diagonal. From eqn. 32 for the i^{th} element obtains: $Y/Y_{0i} = \det[F]/\det[F]_{0i} = f_i$, which means that the 'modes' of the system are completely decoupled and each loop contains exactly one control element[5]. When the modes are decoupled eqn. 31 also yields the same results regardless of the order in which the transistors are reactivated.

Naturally, in general multiple-loop amplifiers the modes are not so neatly isolated; nonetheless some important conclusions can be drawn from the foregoing. For instance, it can be shown[13] that to have the same degree of stability in a multiple-loop case, each characteristic transfer function must satisfy the requirements for a single-loop amplifier. On the one hand, if none of the return-difference loci enters the unit circle centered at the origin of the F-plane then, by eqn. 34, $|\det[F(j\omega)]| \geq 1$ and the closed-loop response is everywhere below the open-loop response. On the other hand, if there is no overshoot, which is usually present in single-loop amplifiers, it does not mean that all $|f_j(j\omega)| \geq 1$, and 'dominant' poles or loops can still exist.

6 The role of dominant poles in assessing stability

The safety margins of the return-ratio against oscillation have been established on the basis of experience and there is no simple experimental way to determine what maximum % change in the value of a parameter is permitted before oscillation sets in, or in what manner the critical point (–1,0) is approached. However, when one of the closed-loop poles dominates (to be defined later) the network response in a frequency interval on the $j\omega$-axis, it is possible to estimate the effect of paramter changes on the movement of this pole and, indirectly, on the transfer characteristic. In the case of a dominant pole the sensitivity is again very closely in agreement with the return-difference in the vicinity of this pole regardless whether the amplifier is of single- or multiple-loop.

In view of the close relationship between sensitivity and predistortion in networks[14], it can be said that, when the feedback loop is closed, the open-loop poles are shifted to the left in the complex p-plane. By assumption, however, the root of det[F] closest to the $j\omega$-axis is a pole, therefore at least one of the open-loop poles must have been shifted to the right by closing the feedback loop. This pole is usually located near the upper edge of the pass band as illustrated in *Figure 6*. In this case det[F] can be closely approximated in the vicinity of this pole by that term of the partial fraction expansion of det[F] in eqn. 33 which contains the dominant root. Clearly the residue of the function at this pole may not be zero or too small. In practical terms this means that an open-loop pole should not be close to this closed-loop pole. For the network determinant $\triangle$ this requires that in all possible network functions the numerator and denominator cannot

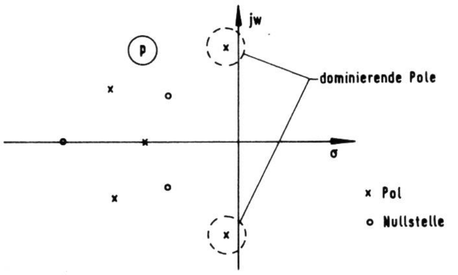

Fig. 6
A possible closed-loop pole-zero distribution
Dominierende Pole – Dominant poles
Pol – Pole
Nullstelle – Zero

have common terms, for otherwise there exist free oscillatory modes which are not detectable at, or not excitable from, certain points in the network. An example of this condition is the single-loop amplifier as it has been defined in Section 4. When the feedback loop is opened, all the modes are controllable (excitable) from the input terminals, but only the meshes or nodes bordering on the input (that part of the network before the first transistor) are observable here. From the output terminals, however, all the modes are observable. When the feedback loop is closed the network is both controllable and observable from every point in the network.

When, e. g. the input node j to the amplifier is excited by a unit current impulse, then a voltage of the form $v(t) = V_0 e^{s_0 t}$ will appear at another node k. If the transfer impedance between these two nodes is $Z_{jk} = \triangle_{jk}/\triangle$, then, apart from some situations that never occur in practice[15], s_0 is a root of $\triangle$ and V_0 is the residue of $\triangle_{jK}/\triangle$ at s_0.

When dominant-pole condition is assumed, the transfer function from e. g. input to any node in the feedback loop can be approximated by

$$\frac{\triangle_{jk}}{\triangle} = \frac{\omega^2_n}{p^2 + \xi\omega_n p + \omega^2_n} \qquad (35a)$$

The ratio of the impulse response at the extreme-value points k half-periods apart yields

$$v(t_o)/v(t_k) = e^{-k \cot \varphi}, \quad k = 0, 1, 2, \ldots \qquad (35b)$$
$$= e^{-\sigma_d T} \qquad k = 2$$

where, from *Figure 7a,* T is the damped frequency period and from *Figure 7b,* $\cos \varphi = \xi$ is the damping ratio which is directly related to the angle φ between the real axis and the radial line to either of the complex poles. The calculation of σ_d and ω_0 should be made from the latest but still accurately observable part of the CRT-trace so that by then the transient components due to the other neglected poles will have died out. In fact, the consistency of the initial portion of the trace with the parameters thus calculated may be used as a measure of the degree of dominance by a given pole. When the transient decays too slowly, to increase the accuracy of the measurement, φ may be calculated from eqn. 35b by using the maxima separated by several periods.

A potentially simpler method of evaluating dominant poles is based on two different definitions of the quality factor Q of the circuit. The first definition generally applicable to any system reads

$$Q = \frac{\text{energy stored}}{\text{energy dissipated per cycle}} \cdot 2\pi$$

From Figure 7a and eqn. 35b

$$Q = \frac{\tfrac{1}{2}Kv(t_o)^2}{\tfrac{1}{2}K\,[v(t_o)^2 - v(t_2)^2]} \cdot 2\pi = \frac{1}{1 - e^{-2\sigma_d T}} \cdot 2\pi \qquad (36a)$$

where K is a constant of the system (e. g. spring constant k for a mechanical system and C or 1/L for an RLC-network). For small σ_d the denominator of eqn. 36a can be approximated by $2\sigma_d T$. The error of this approximation for $2\sigma_d T = 0{,}5$ is ca. 10%. From eqn. 36a

$$Q = 2\pi/2\sigma_d T = \omega_o/2\sigma_d = \tfrac{1}{2}\tan\varphi \qquad (36b)$$

The second definition of Q is based on the frequency selectivity of systems:

$$Q = f_o/(f_2 - f_1), \qquad (37)$$

where f_1 and f_2 are the 'half-power' points of a transfer function compared to its value at f_0 as illustrated in *Figure 8.* For a single-loop amplifier the transfer characteristic between input and output is the most convenient to use for this purpose, given that there is no attempt made outside the feedback loop to equalize the overshoot. For a bandpass amplifier there may be an overshoot at both ends of the pass band.

The optimum synthesis[16] of a second order transfer function realized with normalized-gain single-pole active stages results in a minimum value of $2\sigma_d = 1/Q$ for the return-difference to an active stage. This also determines the least maximum value of the sensitivity attainable for a dominant pole amplifier of arbitrary feedback connections.

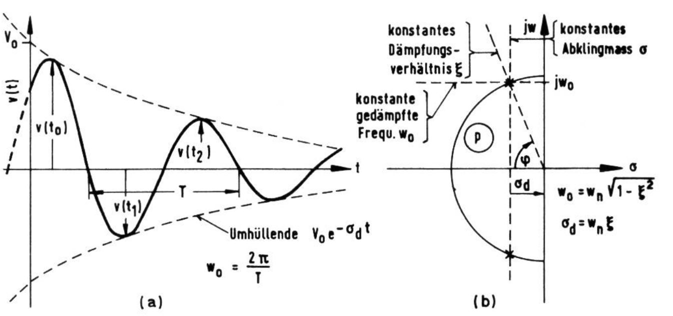

Fig. 7a+b
Impulse response due to dominant poles
Konstantes Dämpfungsverhältnis – Constant damping relation
Konstantes Abklingmass – Constant ratio of decay
Konstante gedämpfte Frequenz – Constant damped frequency
Umhüllende – Enveloped

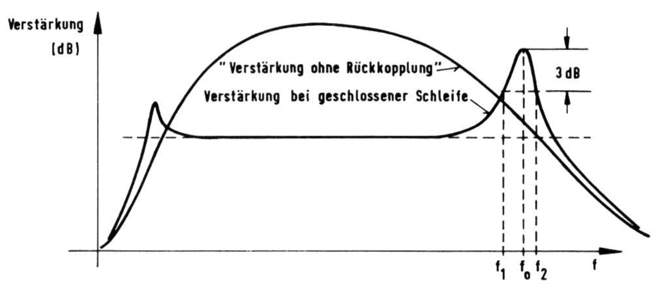

Fig. 8
Typical open- and closed-loop response
Verstärkung – Gain
Verstärkung ohne Rückkoppelung – Gain before feedback
Verstärkung bei geschlossener Schleife – Closed-loop gain

From the frequency-scaling property of linear networks it readily follows that the sum of root sensitivities with respect to all the elements is equal to the root itself[17]. In case of a dominant pole at $p_k = -\sigma_k \pm j\omega_k$, eqn. 15 can be approximated as

$$(p + p_k) \sum_i S^H{}_{g_i} \approx \sum_i g_i \frac{dp_k}{dg_i} = p_k \qquad (38)$$

This equation is particularly useful for optimizing networks to make the dominant pole insensitive to systematic changes in element values, such as those due to temperature variations, aging, etc.[18]. The sum of the sensitivities of all the elements for a dimensionless transfer function is zero. Thus eqn. 38 indicates that the sum of sensitivities of the non-dominant poles which are neglected can be greater than that of the dominant pole. As far as the stability is concerned, the dominant pole approximation is fairly accurate since Re $\{p_k\}$ is very small compared to its imaginary part. When optimizing narrow-band amplifiers or active filters, however, where frequency stability is of equally great concern, the implication of eqn. 38 regarding the neglected poles must be kept in mind.

For single-loop amplifiers with high enough Q the return-difference can be estimated within a few dB's from the amount of overshoot using eqn. 33. For an overshoot of 6 dB in Figure 8 the minimum $|F|$ is about 0.5. For single-loop dominant-pole amplifiers (the majority of practical amplifiers are of this type) it would be more appropriate, instead of the 30° phase- and 10 dB gain-margin, to define a disc of a given radius, e. g. 0.5, centered at (–1,0) as the forbidden region for the return-ratio. The reason for this is that transmission by closed loop changes far more rapidly around f_0 than by open loop. Thus the amount of overshoot is very nearly determined by the minimum approach of the loop-gain curve to the point (–1,0). The phase margin of 30° corresponds to $|F_p| = 0.518$ or 5.7 dB, while the gain margin of 10 dB to $|F_g| = 0.68$. Obviously the weakest point here is the phase margin. A phase margin equivalent to the gain margin is, by elementary calculations, 40°. Since the open-loop gain at the upper band-edge usually decreases monotonically with frequency, to control the overshoot one often encounters practical amplifiers with phase margin sometimes greater than 40° and gain margin less than 10 dB.

7 Application of proposed test-methods to an experimental amplifier

The circuit of the test-amplifier is given in *Figure 9*. Most of the measurements have been performed for three values of the resistor $R_v = 18$, 43, and 75 Ω, which is part of the stabilizing network, resulting in a wide range of phase and gain margins. First the loop-gain was measured, using the 'classical' method to serve as a reference for comparison, by opening the feedback loop at point A and terminating it on the TR3 collector side in 260 Ω normally seen looking into the feedback loop. When an identical amplifier was used to simulate this impedance, the results remained the same. Both the input and output of the amplifier were terminated in their nominal 75 Ω impedance. A signal was fed into the feedback path and the gain and phase around the loop were measured with the Hewlett-Packard Model 8405 A Vector Voltmeter. The results for $R_v = 18$ and 75 Ω are plotted in Figure 11.

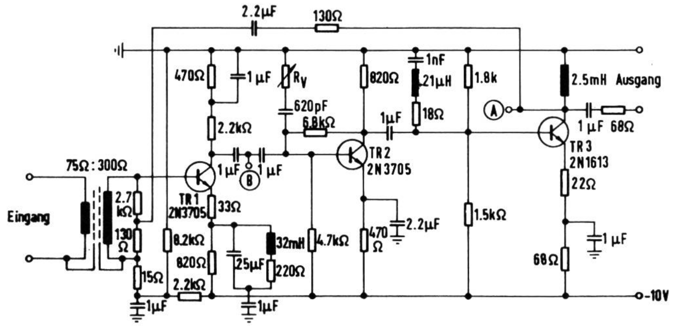

Fig. 9
Circuit diagram of test-amplifier
Eingang – Input
Ausgang – Output

71 Return-difference from driving-point-admittance measurements

This method, discussed in Section 41, is based on admittance measurements and using Blackman's formula given in eqn. 21. The admittance was measured between point B and ground. The amplifier was purposely designed with two capacitors at this point to provide dc isolation. In practical circuits a blocking capacitor may have to be added in series with the point of measurement to leave the biasing intact. In this case the measured susceptance of the blocking capacitor has to be carefully taken into account, particularly at high frequency. The most common methods of admittance measurement are either direct measurement with a bridge or measuring the magnitude and phase of the reflection coefficient. The value of the internal impedance of the measuring instrument is immaterial as long as the amplifier remains linear and does not oscillate when connected to it. The admittance was measured at point B; first under normal operating conditions and then when point A was shorted to ground with a 1 μF capacitor. No measurable difference was observed when, instead of earthing point A, the feedback loop was opened and properly terminated to reduce the loop gain to zero. The results calculated using eqns. 2 and 21 are plotted in Figure 11.

72 Return-difference from transfer-admittance measurement

This method is based on eqn. 22, where F is obtained as the ratio of the results of two measurements. To perform the measurement, a current source has to be simulated. Since current ratios are involved, the actual value of the source current is of no interest as long as it is the same during the normal operation of the amplifier and when the loop is disabled. The experimental arrangement is shown in *Figure 10* using an oscillator and a vector voltmeter. To keep current I_s reasonably constant $|Z|$ must be much greater than $|Z_1|$ for

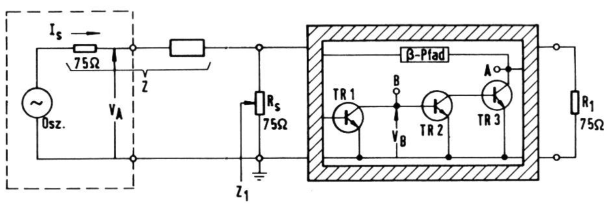

Fig. 10
Test arrangement for measuring transfer-admittance
Pfad – Path
Oszillator – Oscillator
Verstärker – Amplifier

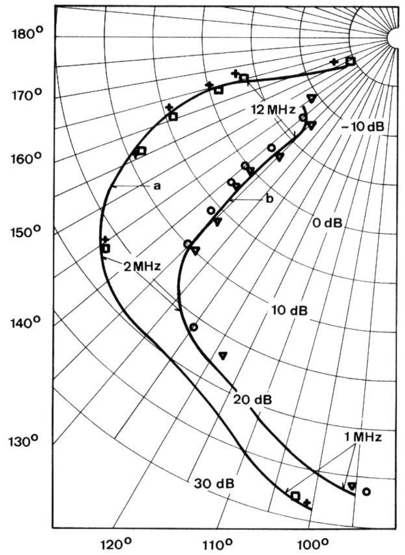

Fig. 11
Loop-gain measurement results
Results with the 'classical method': curve a) for $R_v = 18\,\Omega$
curve b) for $R_v = 75\,\Omega$
Results calculated from driving-point admittances:
+for $R_v = 18\,\Omega$
○for $R_v = 75\,\Omega$
Results calculated from transfer admittances:
□for $R_v = 18\,\Omega$
▽ for $R_v = 75\,\Omega$

both conditions of the loop-gain. In the experiment Z was 2075 Ω. As long as the input impedance to the amplifier has a positive real part, the values of Z_1 are all inside a circle of 75 Ω diameter. Simple analysis shows that the maximum possible current phase variation $\triangle\Theta_{max.}$ between the two conditions of the amplifier

$$\triangle\Theta_{max.} = 2\sin^{-1}\left[\frac{\frac{1}{2}R_s}{|Z + \frac{1}{2}R_s|}\right] \qquad (39a)$$

For Z = 2075Ω, eqn. 39a yields $\triangle\Theta_{max.} = 2.03°$

Similarly, the maximum possible normalized change in the magnitude $|\triangle I_s|_{max.}$ is given by

$$|\triangle I_s/I_s|_{max.} \approx \frac{\frac{1}{2}R_s}{|Z + \frac{1}{2}R_s|} \qquad (39b)$$

which in this case is 3.55%. The error-margins are only slightly larger for complex Z's of the same magnitude. Should the real part of the input-impedance be negative in some frequency range, it could combine with R_s to result in a high impedance in series with Z. This condition can be easily recognized by the amplitude variation of V_A between the two operating conditions or compared to its value at other frequencies.

The transfer-admittance was measured with the probes of the vector voltmeter connected as shown in Figure 10: firstly under normal operating conditions and then with the collector of TR3 earthed. The results calculated using eqn. 22 are plotted in *Figure 11*.

In all cases there is a close agreement between the results obtained by the 'classical' method and those obtained by driving-point- and transfer-admittance measurements. The slight discrepancies can be accounted for by the fact that with the 'classical' method the effect of the internal feedback of a transistor is ignored while here it is largely accounted for in the measurement of Y_n. The method of transfer-admittance measurement is probably the simplest of all since F can be obtained from two vector voltmeter readings by a mere subtraction. Furthermore, with the use of the HP1021A Isolator or the HP11576A 10 : 1 Divider having 100 kΩ and 1 MΩ series resistance respectively, the use of a dc-blocking capacitor, which is a potential source of error, can be avoided.

73 Determination of dominant poles

The experimental arrangement is similar to that of Figure 10 with a pulse generator replacing the oscillator and the response across R_1 observed on a CRT. The resulting transients for $R_V = 18$ and 43 Ω are shown in *Figure 12*. The coordinates of the dominant poles and the corresponding Q's calculated from the traces using eqns. 35b and 36b are listed in *Table I*.

Table I. Dominant poles calculated from Fig. 12a and b

$R_v = 18\,\Omega$				$R_v = 43\,\Omega$			
φ deg.	σ_d rad./sec.	ω_0 rad./sec.	Q	φ deg.	σ_d rad./sec.	ω_0 rad./sec.	Q
82.8	4×10^6	31.8×10^6	3.98	73.	10.1×10^6	$33.\times10^6$	1.6

Another method of determining the dominant poles is to measure the frequency response of the transfer-admittance.

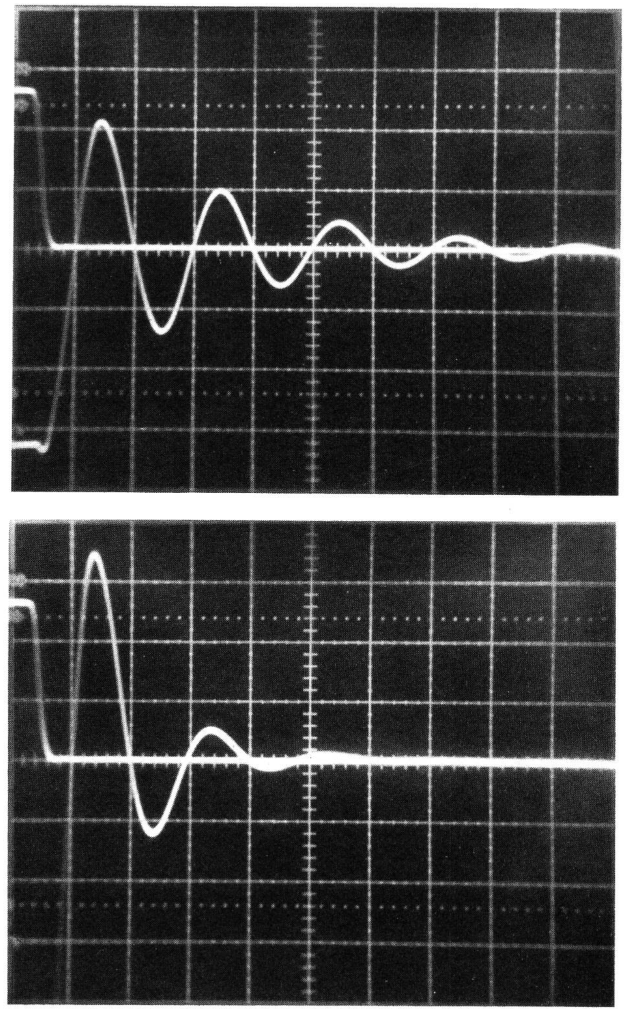

Fig. 12a+b
Responses of test-amplifier to step input
Time scale = 0,1 µs/div.
a) $R_v = 18\,\Omega$ b) $R_v = 43\,\Omega$

The test-arrangement is the same as in Figure 10 except that the output is now measured across R_1. Similarly the 'gain before feedback' was measured by opening the loop at point A and terminating the right terminal of the port thus created in 260 Ω and shorting the left one to earth since the impedance seen at the collector of TR3 is very small. The return-difference F can be calculated as the ratio of the two measurement results since this is just a special case of the transfer-admittance method employed in Section 72. It must be emphasized that the above measurements are not equivalent to measuring insertion gain: The differences can be quite large, particularly for the phase of the transfer function.

The results of the measurements are listed in *Table II*. For $R_v = 18\,\Omega$ the phase shift between the 3 dB-down points is exactly 90°, while the phase of the 'gain before feedback' changes by only 3°, thus confirming the existence of a dominant pole. For $R_v = 43\,\Omega$ the angles are 92° and 6°, respectively. Also the amount of overshoot in the normal state of the amplifier gives the return-difference very accurately. Even for $R_v = 75\,\Omega$ the excess phase is only about 7°, although accurate measurement was difficult due to the flatness of the response curve. The coordinates of the dominant poles can be calculated from eqns. 36b and 37.

74 Input and output admittance measurements

Sometimes these admittances are of interest in finding out how the amplifier might react to changes in the line impedance, in particular to catastrophic line failures (open- or shortcircuit) near the amplifier, although these admittances may not bear any direct relationship to the return-difference and sensitivity as is the case for the amplifier in Figure 9, since shorting the input or output does not reduce the loop-gain to zero.

The input and output admittances were measured on the Wayne Kerr B601 and B801B bridges in their respective frequency bands, while the other side of the amplifier was terminated in 75 Ω. The results are plotted in *Figure 13* A 100 Ω resistor was connected in parallel to the input and output of the amplifier to enable the negative real part of the admittances to be measured. From Figure 13 it is clear that the curve of the input admittance encircles the origin clockwise; therefore the amplifier is open-circuit unstable. At the output the amplifier is conditionally stable since Re $\{Y_{out}\}$ is negative in a frequency band, but $Y(j\omega)$ does not encircle the origin. Both input and output are short-circuit stable.

Termination-dependent stability of linear networks may be discussed briefly as follows: Let *Figure 14* represent an active network describable by four short-circuit admittances. The input admittance at port (1,1') is defined as

$$Y_1 = \frac{I_1}{V_1} = \frac{\Delta}{\Delta_{11}} = Y_{11} - \frac{Y_{12}Y_{21}}{G_2 + Y_{22}} \qquad (40)$$

where Δ is the determinant of the admittance matrix of the network in Figure 11 including G_2 but not G_1. From eqn. 8 the condition for oscillation is obtained

$$G_1 + Y_1 = G_1 + \frac{\Delta}{\Delta_{11}} = 0 \qquad (41)$$

and a similar equation for the output side. For open-circuit stability Δ must not have zeros on the $j\omega$-axis or in the right half plane. Since for the test-amplifier the plot of $Y_1(\pm j\omega)$ twice encircles the origin in the clockwise direction, Δ has two more zeros than Δ_{11} in the right half plane. But the amplifier is short-circuit stable, therfore Δ has exactly one pair of roots here.

The encirclement of the origin clockwise by $Y_1(j\omega)$ is a sufficient but not necessary condition for open-circuit instability. Clearly an amplifier can be open-circuit unstable

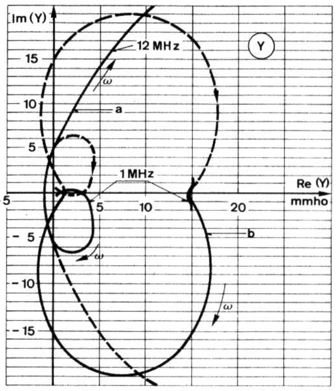

Fig. 13
Results of the input and output admittance measurements:
curve a) is the plot of Y_{in}
curve b) is that of Y_{out}

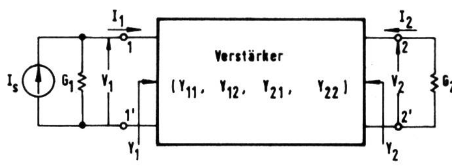

Fig. 14
Representation of a general linear active network

Table II. Gain before feedback and closed-loop response at f_0 and at the 3 dB-down frequencies

	Normal (closed-loop) gain						'Gain-before-feedback'					
MHz	$R_v = 18\,\Omega$		$R_v = 43\,\Omega$		$R_v = 75\,\Omega$		$R_v = 18\,\Omega$		$R_v = 43\,\Omega$		$R_v = 75\,\Omega$	
	dB	deg.	dB	deg.	dB	deg.	dB	deg.	dB	deg.	dB	deg.
1.	–9.6	177	–9.7	178	–9.6	178	18.2	77	17.7	80	14.3	84
1.77	–	–	–	–	–9.0	173	–	–	–	–	8.9	53
3.	–	–	–7.2	162	–	–	–	–	0.	30	–	–
4.2	–.9	138	–	–	–	–	–6.5	10	–	–	–	–
4.73	2.1	94	–	–	–	–	–8.6	9	–	–	–	–
5.	–	–	–4.2	111	–	–	–	–	–8.3	25	–	–
5.3	–	–	–	–	–6.0	121	–	–	–	–	–7.2	34
5.365	–.9	48	–	–	–	–	–10.9	7	–	–	–	–
6.43	–	–	–7.2	70	–	–	–	–	–12.0	24	–	–
8.	–	–	–	–	–9.0	76	–	–	–	–	–12.3	29

when $Y_1(j\omega)$ does not encircle the origin if $\triangle$ and $\triangle_{11}$ have the same number of roots in the right half plane. In this case open-circuit instability implies short-circuit instability. Similarly, the encirclement of the origin counter-clockwise (a very unlikely event in single-loop amplifiers) is a sufficient but not necessary condition for short-circuit instability. When sufficient conditons for open- or short-circuit instability exist, the amplifier may or may not be short- or open-circuit stable.

8 Conclusions

The relationship between feedback and sensitivity has been examined and the sensitivity of a network function presented as a measurable quantity.

Two procedures, both indirect, for evaluating the return-difference without opening the feedback loop have been proposed. The first method, based on Blackman's formula, requires two driving-point admittance measurements at a convenient node in the feedback loop. The admittance is first measured when the amplifier is operating normally and secondly, when the loop is shorted at a remote node in the feedback loop. The return-difference is simply the ratio of these two admittances. This method can be very convenient when admittances are measured directly in polar form.

The second method is based on two transfer-admittance measurements. This procedure requires only a vector voltmeter and a suitable generator. The return-difference can be easily calculated using subtraction only. While with driving-point admittance measurement the determination of the 'reference plane' is problematic because of the sometimes long leads required to make the physical connection from the measuring instrument to the amplifier, with a vector voltmeter no such problem arises when measuring the transfer admittance.

The method of 'dominant poles' can be useful even in the case of multiple-loop amplifiers to determine the minimum value of the return-difference in some loop. The method of transfer-admittance measurement between input and output to determine the dominant poles can be used with single-loop amplifiers, while the step- or impulse-response method is applicable in the multiple-loop case as well, but its use is curtailed by the limited bandwidth of oscillographs.

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