Erratum to 'Rational Lie Algebras and p-Isomorphisms of Nilpotent Groups and Homotopy Types'

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The author has noticed that the proof of Th. 3.2 of [R] contains a flaw. We shall indicate how to modify the argument in [R] so as to set things right.

The trouble occurs on p. 5, 1. 12–13 where it is asserted that the homotopy equivalence $h_0: W \to W$ gives rise to a weak automorphism $\omega: L \to L$ of its associated Lie algebra. However, it may only be asserted that ω is an equivalence in the category in which it lies; it need not be an actual DGL map. To remedy this, we employ a theorem of Quillen ([Q1], [Q2; esp. pp. 263–264] which provides an equivalence between the category of DGL algebras used in [R] and the category whose objects are reduced, rational DGL algebras *which are free as graded Lie algebras* and whose morphisms are homotopy classes (in an appropriate sense) of DGL maps. It is certainly the case that in this latter category, an equivalence always contains a representative ω which is a weak automorphism. To complete the proof of Th. 3.2 of [R], it is only necessary to replace Th. 3.1 of [R] (whose statement and proof are correct, except for a harmless misprint; in formula (3.5), one should have a plus sign rather than a minus sign since $(aa)ab = 2aaaab$, not $-2aaaab$) by the following variant.

THEOREM 3.1′. *There exists a rational, reduced DGL algebra L of finite type, free as a graded Lie algebra, such that any weak automorphism $\omega: L \to L$ is congruent, modulo L^2, to the identity.*

L may be chosen so that $H(L)$ has totally finite dimension, but at the price of possibly foregoing the relation $\omega(x) \equiv x (\mathrm{mod}\, L^2)$ for x of high degree.

Proof. Let L be the free, graded Lie algebra over $\mathbf{Q}$ generated by elements a, b, c, e having degrees 1, 3, 2, 11 respectively and define a differential $d: L \to L$ by setting

$$da = 0, \qquad db = aa, \qquad dc = 0, \qquad de = (ac)ccac + (ac)(ac)ab + (ab)cab.$$

Denote by F the (free) sub-Lie algebra generated by a, b, c. Writing for any weak automorphism $\omega: L \to L$,

$$\omega(a) = ra, \qquad \omega(b) = sb + u(ac), \qquad \omega(c) = tc + v(aa), \qquad \omega(e) = x \cdot e + f,$$

where $r, s, t, u, v, x \in \mathbf{Q}$, $f \in F$, we easily conclude, as in the proof of Th. 3.1 of [R], that $r \neq 0$, $t \neq 0$, $s = r^2$. Further, we obtain $\omega(de) = r^2t^4((ac)ccac) + r^5t^2((ac)(ac)ab) + r^6t((ab)cab) + g \pmod{L^7}$, where g lies in the ideal $I \subset F$ generated by aa. By a straightforward algebraic calculation, one shows that the elements $(ac)ccac$, $(ac)(ac)ab$, $(ab)cab$ are linearly independent modulo I. Then using $\omega(de) = d\omega(e)$, together with $df \varepsilon I$, we conclude that $r^2t^4 = r^5t^2 = r^6t = x$ so that $r = s = t = x = 1$.

To obtain the final assertion of Th. 3.1′, we take any integer $n \geq 11$ and successively attach elements of degree $\geq n+1$ to L so as to kill all homology classes of degree $\geq n$.

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[Q2] ——, *Rational Homotopy Theory*, Ann. of Math. 90 (1969), 205–295.

[R] J. ROITBERG, *Rational Lie Algebras and p-Isomorphisms of Nilpotent Groups and Homotopy Types*, Comment. Math. Helvetici 50 (1975), 1–8.

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