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Factorisation is not unique for higher dimensional knots

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0. Introduction

An n -knot will be a smooth oriented submanifold K of the $(n+2)$ -sphere S^{n+2} , where K is homeomorphic to S^n . A knot is irreducible if it cannot be written as a connected sum of two non-trivial knots. Schubert has shown that every 1-knot can be written uniquely as a connected sum of finitely many irreducible knots (see [S] or [K1, Section 1]). For $n > 2$, Sosinskii has proved that it is still possible to factorise every n -knot into finitely many irreducible knots (cf. [So Theorem 5. 1] or [K1, Section 2]) but Kearton has shown that this factorisation is not necessarily unique for $n = 3$ [K]. In the present note we shall prove the non uniqueness of the factorisation for $(2q-1)$ -knots, $q \geq 3$ and for $(2q)$ -knots, $q \geq 4$.

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1. Factorisation is not necessarily unique for $(2q-1)$ -knots, $q \geq 3$

Let $q \geq 3$ be an integer.

DEFINITION. A *Seifert matrix* A is a square matrix of integers such that $\det(A + (-1)^q A^t) = \pm 1$, where A^t is the transpose of A .

Let A be a non-singular Seifert matrix (that is, $\det(A) \neq 0$). We shall say that A is *irreducible* if A is not S -equivalent to the orthogonal sum of two non-singular Seifert matrices. (See [Le] for the definition of S -equivalence. In the examples that we shall construct, the Seifert matrices will be unimodular, and unimodular Seifert matrices are S -equivalent if and only if they are integrally congruent (see [T, Proposition 4.3])). We shall use the notation

$$S = A + (-1)^q A^t, \quad z = S^{-1}A$$

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LEMMA 1. Let A_1 and A_2 be Seifert matrices with $z_1 = z_2$. Then

$$\begin{pmatrix} A_1 & 0 \\ 0 & -A_1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A_2 & 0 \\ 0 & -A_2 \end{pmatrix}$$

are integrally congruent.

LEMMA 2. There exist irreducible Seifert matrices A_1 and A_2 such that

- a) $z_1 = z_2$
- b) A_1 and A_2 are not S -equivalent
 A_1 and $-A_2$ are not S -equivalent.

(We shall give explicit examples of such Seifert matrices after the proof of this lemma.)

The above two lemmas give the desired result. Indeed, let A_1 and A_2 be Seifert matrices as in Lemma 2.

Levine has shown that the S -equivalence classes of non-singular Seifert matrices correspond biunivoguely to the isotopy classes of simple $(2q-1)$ -knots [Le, Theorems 1, 2, 3]. Note that this implies that irreducible Seifert matrices correspond to irreducible knots. Let K_1, L_1, K_2, L_2 be the simple $(2q-1)$ -knots corresponding to $A_1, -A_1, A_2, -A_2$ respectively. These knots are irreducible, because A_1 and A_2 are irreducible. By Lemma 1,

$$\begin{pmatrix} A_1 & 0 \\ 0 & -A_1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A_2 & 0 \\ 0 & -A_2 \end{pmatrix}$$

are integrally congruent, as $z_1 = z_2$. Therefore they are S -equivalent. So by [Le, Theorem 3] the connected sum of K_1 and L_1 is isotopic to the connected sum of K_2 and L_2 . On the other hand, [Le, Theorem 1] shows that K_1 is not isotopic either to K_2 or to L_2 , as the Seifert matrices are not S -equivalent.

Proof of Lemma 1. Let A be a Seifert matrix, and let

$$M_1 = \begin{pmatrix} A & 0 \\ 0 & -A \end{pmatrix}, \quad M_2 = \begin{pmatrix} 0 & (-1)^q(1-z^t) \\ z & 0 \end{pmatrix}.$$

Then M_1 and M_2 are integrally congruent. Indeed, let

$$X = \begin{pmatrix} 1-z & (-1)^q S^{-1} \\ -z & (-1)^q S^{-1} \end{pmatrix} = \begin{pmatrix} I & I \\ 0 & I \end{pmatrix} \begin{pmatrix} I & 0 \\ 0 & (S^{-1})^t \end{pmatrix} \begin{pmatrix} I & 0 \\ (-1)^{q+1} A & I \end{pmatrix}$$

where I is the identity matrix.

One checks by direct computation that $M_2 = X^t M_1 X$ (it is useful to note that $1 - z^t = AS^{-1}$, and that $(1 - z^t)A = Az$). This proves Lemma 1, as $z_1 = z_2 = z$,

$$\begin{pmatrix} A_1 & 0 \\ 0 & -A_1 \end{pmatrix} \text{ and } \begin{pmatrix} A_2 & 0 \\ 0 & -A_2 \end{pmatrix}$$

are both congruent to

$$\begin{pmatrix} 0 & (-1)^q (1 - z^t) \\ z & 0 \end{pmatrix}.$$

Proof of Lemma 2. Let ϕ be the cyclotomic polynomial corresponding to the 15th roots of unity. Let t be the Jordan matrix associated with ϕ , and let $z = (1 - t)^{-1}$. Note that $\det(z) = 1$: indeed, $\det(1 - t) = \phi(1) = 1$.

Let ζ be a primitive 15th root of unity. Sending ζ to ζ^{-1} induces a non-trivial involution on $\mathbf{Z}[\zeta]$. We shall denote this involution by an overbar.

Let $\Delta = \{x \in \mathbf{Q}(\zeta) \mid \text{Tr}_{\mathbf{Q}(\zeta)/\mathbf{Q}}(x\mathbf{Z}[\zeta]) \subset \mathbf{Z}\}$ be the inverse different of the extension $\mathbf{Q}(\zeta)/\mathbf{Q}$. We have: $\bar{\Delta} = \Delta$.

DEFINITION. Let V be a torsion free $\mathbf{Z}[\zeta]$ -module of finite rank. We shall say that a hermitian or skewhermitian form

$$h : V \times V \rightarrow \Delta$$

is *unimodular*, if

$$\begin{aligned} \text{ad}(h) : V &\rightarrow \text{Hom}_{\mathbf{Z}[\zeta]}(V, \Delta) \\ x &\rightarrow h(\cdot, x) \end{aligned}$$

is an isomorphism.

Claim 1. The integral congruence classes of Seifert matrices A such that

$$(A + (-1)^q A^t)^{-1} A = z \tag{1}$$

($z = (1-t)^{-1}$ as above, fixed; q a fixed integer) are in bijection with the isometry classes of $(-1)^q$ -hermitian unimodular forms

$$h: \mathbf{Z}[\zeta] \times \mathbf{Z}[\zeta] \rightarrow \Delta.$$

Proof of Claim 1. Let A be a Seifert matrix with property (1), and let $S = A + (-1)^q A^t$. Rank $(A) = \text{degree}(\phi) = 8$. Let V be a free \mathbf{Z} -module of rank 8. We can consider S as a $(-1)^q$ -symmetric form $S: V \times V \rightarrow \mathbf{Z}$, and $t = 1 - z^{-1}: V \rightarrow V$ will be an isometry for S .

Setting $\zeta \cdot v = t(v)$ for v in V makes V into a $\mathbf{Z}[\zeta]$ -module. As t corresponds to the Jordan matrix of ϕ , V is isomorphic to $\mathbf{Z}[\zeta]$.

As in [B-M, §1], we associate to S a $(-1)^q$ -hermitian form

$$h: \mathbf{Z}[\zeta] \times \mathbf{Z}[\zeta] \rightarrow \Delta$$

such that

$$\text{Tr}_{\Phi(\zeta)/\Phi} h(\alpha x, y) = S(\alpha x, y) \quad \forall \alpha \in \Phi(\zeta) \quad \forall x, y \in V. \quad (2)$$

It is easy to check that h is unimodular and that congruent Seifert matrices determine isometric $(-1)^q$ -hermitian forms.

Conversely, given a unimodular $(-1)^q$ -hermitian form $h: \mathbf{Z}[\zeta] \times \mathbf{Z}[\zeta] \rightarrow \Delta$, the formula (2) determines a $(-1)^q$ -symmetric matrix S such that $\det(S) = \pm 1$ and t is an isometry for S . Set $A = Sz$. Then $A + (-1)^q A^t = S$, therefore A is a Seifert matrix satisfying (1). Isometric $(-1)^q$ -hermitian forms determine congruent Seifert matrices.

Claim 2. The isometry classes of unimodular $(-1)^q$ -hermitian forms

$$h: \mathbf{Z}[\zeta] \times \mathbf{Z}[\zeta] \rightarrow \Delta$$

are in bijection with $U_0/N(U)$, where U is the group of units of $\mathbf{Z}[\zeta]$, U_0 is the group of units of $\mathbf{Z}[\zeta + \bar{\zeta}]$, and $N: U \rightarrow U_0$, $N(u) = u\bar{u}$, is the norm map.

Proof of Claim 2. Let g be the minimal polynomial of $\zeta + \bar{\zeta}$, and let

$$\alpha_0 = \frac{1}{g'(\zeta + \bar{\zeta})} \frac{1}{\zeta - \bar{\zeta}}.$$

Let Δ_1 be the inverse different of the extension $\mathbf{Q}(\zeta)/\mathbf{Q}(\zeta + \bar{\zeta})$, and Δ_2 the inverse different of $\mathbf{Q}(\zeta + \bar{\zeta})/\mathbf{Q}$. Then $\Delta = \Delta_1 \cdot \Delta_2$ [L, III. §1, Proposition 5] and

$$\Delta_1 = \frac{1}{\zeta - \bar{\zeta}} \mathbf{Z}[\zeta]$$

$$\Delta_2 = \frac{1}{g'(\zeta + \bar{\zeta})} \mathbf{Z}[\zeta + \bar{\zeta}]$$

[L, III. §1 Corollary of Proposition 2]. Therefore $\Delta = a_0 \mathbf{Z}[\zeta]$. Notice that $\bar{a}_0 = -a_0$.

Let $h: \mathbf{Z}[\zeta] \times \mathbf{Z}[\zeta] \rightarrow \Delta$ be a unimodular $(-1)^q$ -hermitian form. We have: $h(x, y) = ax\bar{y}$ for some a in Δ such that $\bar{a} = (-1)^q a$.

As we can identify $\text{Hom}_{\mathbf{Z}[\zeta]}(\mathbf{Z}[\zeta], \Delta)$ with Δ , the unimodularity of h implies that $a\mathbf{Z}[\zeta] = \Delta$. Therefore $a\mathbf{Z}[\zeta] = a_0\mathbf{Z}[\zeta]$. This implies that aa_0^{-1} is a unit. We have $\overline{aa_0^{-1}} = (-1)^{q+1}aa_0^{-1}$.

Set

$$u = \begin{cases} aa_0^{-1} & \text{if } q \text{ is odd} \\ aa_0^{-1}(\zeta - \bar{\zeta}) & \text{if } q \text{ is even} \end{cases} \quad (3)$$

$\zeta - \bar{\zeta}$ is a unit: $(\zeta - \bar{\zeta})^2(\zeta + \bar{\zeta})(-\zeta - \bar{\zeta} + 1) = 1$.

Therefore u is in U_0 in both cases. Conversely, to $u \in U_0$ we associate the $(-1)^q$ -hermitian form $h(x, y) = ax\bar{y}$ where a is given by (3). One checks easily that two $(-1)^q$ -hermitian forms are isometric if and only if the corresponding units are in the same class in $U_0/N(U)$.

Let us determine the cardinality of $U_0/N(U)$. We have

$$[U_0 : N(U)] = \frac{[U_0 : U_0^2]}{[N(U) : U_0^2]}.$$

Using the theorem of Dirichlet on the rank of the group of units, we see that $[U_0 : U_0^2] = 16$.

Let μ be the group of roots of unity in $\mathbf{Q}(\zeta)$. Then

$$[N(U) : U_0^2] = [U : \mu U_0] = Q$$

and $Q = 2$ [L1, Chap. 3, Theorem 4.1]. So $[U_0 : N(U)] = 8$. (We shall actually exhibit 8 distinct classes of $U_0/N(U)$ in the next section.)

Applying Claim 1 and Claim 2, we see that there are 8 non-congruent Seifert matrices A such that

$$(A + (-1)^q A')^{-1} A = z. \quad (1)$$

Therefore it is possible to choose A_1 and A_2 satisfying (1), and such that A_1 is not congruent either to A_2 or to $-A_2$. But congruence and S -equivalence are the same in this case, because the Seifert matrices are unimodular (see [T, Proposition 4.3]). A_1 and A_2 are irreducible, as their Alexander polynomial is irreducible.

Explicit examples

Let $\zeta = e^{2i\pi/15}$, and let $u_1 = 1$, $u_2 = \zeta + \zeta^{-1}$. We have $u_2(-u_2^3 + u_2^2 + 4u_2 - 4) = 1$, therefore u_2 is a unit. But u_2 is not in $N(U)$: indeed, u_2 is conjugate to $\zeta^7 + \zeta^{-7}$ which is negative. Clearly $-u_2$ is also negative, therefore not in $N(U)$. Using similar methods for the units $u_3 = \zeta^2 + \zeta^{-2}$, $u_4 = u_2 u_3 = \zeta + \zeta^{-1} + \zeta^3 + \zeta^{-3}$, we see that $u_1, -u_1, u_2, -u_2, u_3, -u_3, u_4, -u_4$ are all in different classes of $U_0/N(U)$. In the proof of Lemma 2 we have seen that the cardinality of $U_0/N(U)$ is 8, therefore we have a complete set of representants of $U_0/N(U)$.

Using the method given in the proof of Lemma 2, let us associate the Seifert matrices A_i to the units u_i , $i = 1 \cdots 4$.

Then the $\begin{pmatrix} A_i & 0 \\ 0 & -A_i \end{pmatrix}$ are all different factorisations of the same Seifert matrix B (see Lemma 1). Moreover, B has no other factorisations than these four. Direct computation gives the following matrices for A_1 and A_2 :

q odd:

$$A_1 = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & -1 & -2 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ -1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 1 & 1 & 1 & 1 \\ -2 & -2 & -1 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 2 & 2 & 2 & 2 & 1 & 0 & -2 & -3 \\ 2 & 2 & 2 & 2 & 2 & 1 & 0 & -2 \\ 2 & 2 & 2 & 2 & 2 & 2 & 1 & 0 \\ 1 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\ 0 & 1 & 2 & 2 & 2 & 2 & 2 & 2 \\ -2 & 0 & 1 & 2 & 2 & 2 & 2 & 2 \\ -3 & -2 & 0 & 1 & 2 & 2 & 2 & 2 \\ -4 & -3 & -2 & 0 & 1 & 2 & 2 & 2 \end{pmatrix}$$

q even

$$A_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & -1 & -2 & -2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} 0 & 0 & 0 & -1 & -2 & -3 & -3 & -2 \\ 0 & 0 & 0 & 0 & -1 & -2 & -3 & -3 \\ 1 & 0 & 0 & 0 & 0 & -1 & -2 & -3 \\ 2 & 1 & 0 & 0 & 0 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & -1 \\ 3 & 3 & 2 & 1 & 0 & 0 & 0 & 0 \\ 2 & 3 & 3 & 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 3 & 3 & 2 & 1 & 0 & 0 \end{pmatrix}$$

2. Factorisation is not necessarily unique for $(2q)$ -knots, $q \geq 4$

Let $q \geq 4$ be an integer. Let $\Lambda = \mathbf{Z}[t, t^{-1}]$, and let T be a finitely generated \mathbf{Z} -torsion Λ -module such that $(1-t): T \rightarrow T$ is an isomorphism.

DEFINITION. $L: T \times T \rightarrow \mathbf{Q}/\mathbf{Z}$ is a *Levine pairing* if L is \mathbf{Z} -bilinear, non-singular, $(-1)^{q+1}$ -symmetric, such that

$$L(tx, ty) = L(x, y) \quad \text{for } x, y \text{ in } T.$$

In [Le 1] Levine associates to every $(2q)$ -knot K a Levine pairing on the \mathbf{Z} -torsion part T of $H_q(\tilde{X})$, \tilde{X} being the maximal abelian cover of $X = S^{2q+2} \setminus K$. Isotopic knots have isometric pairings. He also shows that every Levine pairing can be realized by a simple $(2q)$ -knot [Le 1, Theorem 13.1]. Conversely, Kojima has shown that if $H_q(\tilde{X})$ is finite and 2-torsion free and if $q \geq 4$, then simple $(2q)$ -knots having isometric Levine pairings are isotopic [Ko, Theorem 1]. Therefore, the following examples determine simple $(2q)$ -knots which factorise in more than one way:

q odd

Let $T = \mathbf{Z}/5$, and let $t(x) = -x$ for x in T . Then $L_1, L_2: T \times T \rightarrow \mathbf{Q}/\mathbf{Z}$ given by $L_1(x, y) = \frac{1}{5}xy$, $L_2(x, y) = \frac{2}{5}xy$ are Levine pairings. Clearly L_1 is not isometric either to L_2 or to $-L_2$. But

$$\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1/5 & 0 \\ 0 & 4/5 \end{pmatrix} \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1/5 \\ 1/5 & 0 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 2/5 & 0 \\ 0 & 3/5 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 1 & 1 \end{pmatrix},$$

and the isomorphisms obviously commute with $t \oplus t$.

q even

Let $T = \mathbf{Z}/5 \oplus \mathbf{Z}/5$, $t: T \rightarrow T$ given by the matrix

$$\begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}$$

and let $L_1, L_2: T \times T \rightarrow \mathbf{Q}/\mathbf{Z}$ be the Levine pairings given by the matrices

$$\begin{pmatrix} 0 & 1/5 \\ 4/5 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 & 2/5 \\ 3/5 & 0 \end{pmatrix}$$

respectively.

L_1 is not Λ -isometric either to L_2 or to $-L_2$. Indeed, suppose that L_1 is Λ -isometric to $\varepsilon \cdot L_2$, for $\varepsilon = +1$ or -1 , and let X be the matrix corresponding to this isometry. Then $\det(X) = 2\varepsilon$.

Let

$$X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

The relation $tX = Xt$ implies $\det(X) = (a+b)^2$. But $(a+b)^2 = 2\varepsilon$ is impossible. $L_1 \oplus -L_1$ and $L_2 \oplus -L_2$ are both Λ -isometric to

$$\begin{pmatrix} 0 & 0 & 0 & 2/5 \\ 0 & 0 & 3/5 & 0 \\ 0 & 2/5 & 0 & 0 \\ 3/5 & 0 & 0 & 0 \end{pmatrix}$$

the isometries are given by

$$\begin{pmatrix} I & I \\ -I & I \end{pmatrix} = \begin{pmatrix} I & I \\ 0 & I \end{pmatrix} \begin{pmatrix} I & 0 \\ 2I & I \end{pmatrix} \begin{pmatrix} 2I & 0 \\ 0 & I \end{pmatrix}$$

and

$$\begin{pmatrix} 3I & I \\ 2I & I \end{pmatrix} = \begin{pmatrix} I & I \\ 0 & I \end{pmatrix} \begin{pmatrix} I & 0 \\ 2I & I \end{pmatrix}$$

respectively, with $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

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