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Parallelizability of Finite H-Spaces

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This note is a contribution to the study of how closely topological spaces with a multiplication (*H*-spaces) geometrically resemble Lie groups.

THEOREM. *Let X be a finite H -space and suppose that $X_{(2)}$ has a factor which at the prime 2 is S^7 , or a Lie group and that*

- 1) X is simply-connected, or
- 2) $\pi_1 X$ is a odd p -group, or
- 3) $\pi_1 X$ is infinite and has no 2-torsion.

Then X has the homotopy type of a closed parallelizable smooth manifold.

For a history and for our other related results see [2]. For instance, without the assumption on $X_{(2)}$ it is shown there that X is homotopy equivalent to a closed manifold. All known examples of finite *H*-spaces in fact are products of (just) S^7 , RP^7 and Lie groups at the prime 2, so that our theorem for the given fundamental groups includes all the known examples. Here, we just prove the theorem in case $\pi_1 X = 0$; the non-simply connected cases are readily dealt with using similar ideas together with methods of handling fundamental group difficulties developed in [2].

Notice that S^7 is the two-fold cover of RP^7 , and every Lie group is the two-fold cover of a closed manifold [2, Lemma 2.1]. Thus $X_{(2)}$ is the two-fold cover of the localization at 2 of a finite complex Y . There is a map $Y \rightarrow X_{(0)}$ so that the composite $X_{(2)} \rightarrow Y_{(2)} \rightarrow X_{(0)}$ is just localization. Let Z be the homotopy pullback:

$$\begin{array}{ccc} Z & \longrightarrow & X[\frac{1}{2}] \\ \downarrow & & \downarrow \\ Y_{(2)} & \longrightarrow & X_{(0)} \end{array}$$

Observation 1. Z is a finite Poincaré complex with 2-fold cover homotopy equivalent to X .

Observation 2. There is a map $Z \rightarrow BO$ lifting the Spivak normal fibration such that $X \rightarrow Z \rightarrow BO$ is nullhomotopic.

For the proofs of related statements see [3]. Now, consider a degree-one normal map $f: M \rightarrow Z$ associated to the lifting given by (2), and more importantly, the surgery obstruction of the two-fold cover $\tilde{f}: \tilde{M} \rightarrow X$. Note that \tilde{f} is covered by trivial bundle data. The only nonzero elements in $L_n(0)$ in the image of the transfer occur for $n \equiv 0 \pmod{4}$ and are detected by signature. Now $\text{sign}(\tilde{M}) = 0$ by the Hirzebruch signature formula and the stable parallelizability of M and $\text{sign}(X) = 0$ by an easy application of the Milnor–Moore theorem on Hopf algebras (e.g. X is rationally a product of odd spheres) so that this obstruction vanishes as well. Thus, we can complete surgery on \tilde{f} producing a stably parallelizable manifold N homotopy equivalent to X . Except in low dimensions where parallelizability is automatic, the obstructions to N being parallelizable are in fact just the Euler characteristic and, if the dimension is $\equiv 1 \pmod{2}$, the $Z/2Z$ semicharacteristic, (see e.g. [1]), but these vanish for N since they obviously do for X . Q.E.D.

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