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Autor: Dosch, Hans Günther

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THE CONCEPT OF SIGN AND SYMBOL IN THE WORK OF HERMANN HELMHOLTZ AND HEINRICH HERTZ

The application of the concept of sign and symbol in the scientific work of Helmholtz and Hertz is reported for two typical cases: the theory of harmony and disharmony of Helmholtz and the elimination of «force» in Hertz's mechanics. This feature is put in the context of development of physics from Newton to modern times.

On peut rendre compte de l'application du concept de signe et de symbole dans le travail scientifique de Helmholtz et Hertz par deux cas typiques : la théorie de l'harmonie et de la disharmonie de Helmholtz et l'élimination de la «force» dans la mécanique de Hertz. Cette particularité est située dans le contexte du développement de la physique de Newton aux temps modernes.

There is no comprehensive essay not to speak of a monography on the theory of signs by Helmholtz and Hertz, there are rather some scattered remarks on their concept in their scientific work. Nevertheless Cassirer refers emphatically to these authors when it comes to this subject. In his work *Substanzbegriff und Funktionsbegriff*[1] Cassirer puts the naive *Dingbegriff* [concept of thing] against the concept of sign and refers explicitely to Helmholtz' *Zeichentheorie*: Exact science, even if it sticks to the concept of thing, can give it sense only through the purely formal relations on which the connection with experience rests. «Especially pregnant is this treat in the theory of signs of Helmholtz, which is a characteristic and typical form of scientific epistemology»[2]. It follows an extensive quotation from Helmholtz to the content of which I shall come back later. In the

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1. Ernst CASSIRER, *Substanzbegriff und Funktionsbegriff*, Berlin : B. Cassirer, 1910.
2. l. c., p. 404.

first volume of *Die Philosophie der symbolischen Formen*[3] he refers again to exact science and states that mathematical and physical science were the first to be conscious of the symbolic character of their scientific means[4]:

> Es ist insbesondere die mathematisch-physikalische Erkenntnis gewesen, die sich dieses Symbolcharakters ihrer Grundmittel am frühesten und schärfsten bewusst geworden ist.

There he stresses the creative act of forming the symbol and refers consequently rather to Hertz than to Helmholtz, again with extensive quotations from the former.

One may wonder why Cassirer stresses the importance of these in comparison with a philosophical theory of signs and symbols rather unsystematic remarks ; I think the resolution lies in the following : Neither in *Substanzbegriff und Funktionsbegriff* nor in *Die Philosophie der symbolischen Formen* Cassirer wanted to create a completely new scheme, according to which one had to proceed, but he was rather looking for the scheme in which cultural activities can fit. This is very clearly expressed in his essay on *Determinismus und Indeterminismus in der modernen Physik*[5]. There he notices that though the epistemological background of different scientists might vary considerably, their scientific conclusions are often very close and the process of scientific advance has not been inhibited. He concludes[6]:

> Auch wir müssen mit diesem Tun beginnen und es unmittelbar befragen. Wenn wir, statt den Theorien über die Physik vielmehr dem Prozess der physikalischen Begriffsbildung selbst folgen, so werden wir vielleicht erwarten und hoffen dürfen, in ihm gewisse Grundbestimmungen aufzufinden, die gegenüber dem Wechsel der verschiedenen erkenntnistheoretischen Bezugssysteme invariant sind.

In this sense I as a scientist see the task of my contribution to shed some light on Helmholtz' and Hertz' concepts of sign and symbol in the *context of their scientific work*.

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3. Ernst CASSIRER, *Die Philosophie der symbolischen Formen*, Berlin : B. Cassirer, 1923.
4. l. c., p. 5.
5. Ernst CASSIRER, *Determinismus und Indeterminismus in der modernen Physik*, Göteborg: 1937, reprinted in *Zur modernen Physik*, Darmstadt : Wiss. Buchgesellschaft, 1980.
6. l. c., p. 162.

There can be no doubt that the concept of sign in Helmholtz' epistemological scheme has its origin in physiology, and especially in the law of specific sensory energy of his teacher Johannes Müller. He states, near the end of his life, explicitely[7]:

> Dass wir ein Object als farbiges Gesichtsbild sehen, hängt nur vom Auge ab; in welcher besonderen Farbe wir es sehen, allerdings auch von der Art des Lichtes, das es uns zusendet. Dies Gesetz ist von Johannes Müller, dem Physiologen, nachgewiesen worden und als das *Gesetz der spezifischen Sinnesenergien* bezeichnet ...
>
> Ich habe deshalb die Beziehung zwischen der Empfindung und ihrem Objekte so zu formulieren zu müssen geglaubt, dass ich die Empfindung nur für ein *Zeichen* von der Einwirkung des Objectes erklärte. Zum Wesen des Zeichens gehört nur, dass für das gleiche Object immer dasselbe gegeben werde. Uebrigens ist gar keine Art von Aehnlichkeit zwischen ihm und seinem Object nötig ...

It is therefore not astonishing that most of the references to his concept of signs are to be found in his monumental *Handbuch der Physiologischen Optik*[8] and there in section 3, « Die Lehre von den Gesichtswahrnehmungen ».

From there Cassirer quotes and paraphrases longer passages in his work *Substanzbegriff und Funktionsbegriff*. I shall also quote a decisive piece[9]:

> Insofern die Qualität unserer Empfindungen uns von der Eigenthümlichkeit der äusseren Einwirkung, durch welche sie erregt ist, eine Nachricht giebt, kann sie als ein Zeichen derselben gelten, aber nicht als ein Abbild. Denn von einem Bilde verlangt man irgend eine Art der Gleichheit mit dem abgebildeten Gegenstande. ... Ein Zeichen aber braucht gar keine Art der Aehnlichkeit mit dem zu haben, dessen Zeichen es ist. Die Beziehung zwischen beiden beschränkt sich darauf, dass das gleiche Objekt unter gleichen Umständen zur Einwirkung kommend, das gleiche Zeichen hervorruft, und dass also ungleiche Zeichen immer ungleicher Einwirkung entsprechen.
>
> Der populären Meinung gegenüber, welche auf Treue und Glauben die volle Wahrheit der Bilder annimmt, ... mag dieser Rest von Aehnlichkeit ... sehr gering erscheinen. In Wahrheit ist er es nicht; denn damit kann noch eine Sache von allergrösster

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7. Hermann HELMHOLTZ, *Goethes Vorahnung kommender Naturwissenschaftlicher Ideen*, Berlin : Gebr. Paetel, 1892, p. 45 f.

8. Hermann HELMHOLTZ, *Handbuch der Physiologischen Optik*, Hamburg und Leipzig : 2. Aufl., 1896.

9. l. c., p. 586.

> Tragweite geleistet werden, nämlich die Abbildung der Gesetzmässigkeit. . . . Da Gleiches in unsere Empfindungswelt durch gleiche Zeichen angezeigt wird, so wird der naturgesetzlichen Folge gleicher Wirkungen auf gleiche Ursachen auch eine ebenso regelmässige Folge im Gebiete unserer Empfindungen entsprechen.
> Wenn also unsere Sinnesempfindungen in ihrer Qualität auch nur Zeichen sind, . . . so sind sie doch nicht als leerer Schein zu verwerfen, sondern sie sind eben Zeichen von etwas, . . . und was das Wichtigste ist, das Gesetz dieses Geschehens können sie uns abbilden.

It is clear that Cassirer ist most fascinated by the emphasis which is put by Helmholtz on the lawful relation between the signs. As is well known Cassirer continues the thoughts of Helmholtz and ends with the statement that the subjects of physics in their lawful connection are not signs of something objective but objective signs which satisfy certain conceptual conditions and postulates[10]. In this respect he is nearer to the concept of sign of Hertz than that of Helmholtz, but before I come to that, let me shortly discuss the imput of the sign theory on Helmholtz' scientific work.

As mentioned above, he introduces his concept of sign in the context of physiological optics, but I want to discuss its implicit application in physiological acoustics, namely in his famous theory of harmony and disharmony in musics. Since this is one of the oldest scientific problems in the european culture, brought to our attention by Pythagoras and his school, it might be an appropriate interlude for a colloquium on science and culture.

As it is well known, Pythagoras discovered that the harmonic intervals, *συμφωνίαι* are related to *λόγοι επιμόριοι* of the sacred tetrakys :

2:1	octave
3:2	fifth
4:3	fourth

Scientists and philosophers since the Pythagorean times — Klaudios Ptolemaios, Descartes, Leibniz, Euler, to name only the most famous ones — have tried to find an explanation for that close connection between mathematics and harmony, but none was recognized as satisfactory.

10. E. Cassirer, *Substanzbegriff...*, p. 405.

The first real step to an explanation was made by Rameau and d'Alembert. D'Alembert writes[11]:

> Si on fait resonner un corps sonore, on entend, outre le son principal & son octave, deux autres sons très-aigus, dont l'un est la douzième au-dessus du son principal [...] & l'autre est la dix-septieme majeure audessus de ce même son.

This observation was the basis of Rameau's theory de la *basse fondamentale*, and it was assumed that *sol* and *mi* above the *ut (do)* form «l'accord le plus parfait [...], puisque cet accord est l'ouvrage de la nature[12].»

D'Alembert knew of course that the octave corresponded to a tone with double frequency of the fundamental one, the twelth with triple frequency and so on. They are called the higher harmonics of the fundamental tone. It was assumed that the time dependence of the pressure vibrations of the air corresponding to a tone with definite pitch were sinuoidal, i.e. as the amplitudes of a swinging pendulum.

Though this was an important step in the theory of harmony it was certainly not a scientific explanation. Furthermore, if you hear a sound, it is by no means so easy for the unprepared listener to hear the «other tones» in the «principal tone» and furthermore it was somewhat unclear what it meant that one tone contained other ones. The problem of the compositnes of tones became even more acute after the invention of the siren which allowed sytematic studies and opened a long debate over the question of the essence of a tone *(Wesen des Tones)*. Interpreting experiments of August Seebeck[13], Georg Simon Ohm[14] concluded that the ear in hearing a tone decomposed it in simple harmonics, each corresponding to a sinuoidal vibration of the air. The lowest component determines the pitch of the tone. In that way he introduced a new mathematical tool into acoustics, the Fourier analysis. Against the interpretation of Ohm Seebeck[15] was holding that the computed intensity of the fundamental harmonic component was in some cases much too low in order to account for the pitch. Seebeck concluded that the periodicity was essential for the tone, and not

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11. D'ALEMBERT, *Élémens de Musique, Théorique et Practique, Suivant les Principes de M. Rameau*, Paris: 1752, p. 12.
12. l. c., p. 18.
13. Georg S. OHM, *Annalen der Physik und Chemie*, vol. LIII (1841), p. 417 ff.
14. l. c., vol. LIX (1843), p. 513 ff.
15. l. c., vol. LX (1843), p. 449.

its lowest harmonic. This was generally accepted at the time, and the dispute between Ohm and Seebeck seemed to be settled in favour of the later.

Helmholtz was taking up the subject some ten years later, and in his *Lehre von den Tonempfindungen*[16] he devotes a long section to the decomposition of sounds *(Klänge)* by the ear. In his epistemological scheme it was clear that he was seeking the « signs » from which we construct the sound and as those he identified the harmonics of Ohm. He writes[17]:

> Die Obertöne sind nämlich ein Phänomen, welches der reinen *Empfindung* des Ohres angehört; die Zusammenfassung einer Reihe von Partialtönen zu einem Klang . . . ist ein Vorgang, welcher in das Gebiet nicht der Empfindungen, sondern der *Wahrnehmungen* fällt. *Empfindungen* nennen wir die Eindrücke auf unsere Sinne, insofern sie uns nur als Zustände unseres Körpers . . . zum Bewusstsein kommen; *Wahrnehmungen*, insofern wir aus ihnen uns die Vorstellung äusserer Objekte bilden.

In the fourth edition of his *Lehre von den Tonempfindungen* he is less pictorial and he introduces « perception » and « apperception[18] », but at any rate it is clear that the perceptions of the harmonics are the relevant signs of a complex tone, i.e. if we seek lawful connections in physiological acoustics, we have to recourse to these. He thus formulated what is known as Ohms Law of acoustics[19]:

> The human ear perceives only pendular (i.e. sinusoidal) vibrations of the air as a simple tone and resolves all other periodic motions of the air into a series of pendular vibrations and perceives the series corresponding to these tones.

The pitch of a complex tone (musical sound) is determined by the slowest pendular vibration, the timbre by the admixture of the faster ones (higher harmonics). The mathematical description of this decomposition is the Fourier analysis. One can e. g. introduce with its help a distance in « timbre space » which agrees astonishingly well with psychoacoustic findings[20]. The solution of the

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16. Hermann HELMHOLTZ, *Die Lehre von den Tonempfindungen als physiologische Grundlage für die Theorie der Musik*, Braunschweig: Vieweg, 1865.
17. 1. c., 2nd ed., Braunschweig: 1865, p. 101.
18. 1. c., 4th ed., Braunschweig :1877, p. 107.
19. 1. c., p. 97.
20. s. R. PLOMB, *Aspects of tone sensation*, London: Academic Press, 1976, ch. 6.

old problem of harmony is now remarkably simple: one only needs the psychoacoustic fact that two simple tones (i.e. those corresponding to pendular vibrations) which differ little in frequency make the impression of roughness, something like a flickering light. The most perfect harmony, the octave, has a frequency ratio 2:1, and in this case the analysis yields very simply, that none of the higher harmonics come close enough in frequency in order to create the sensation of roughness quoted above. On the other hand the most dissonant intervals, the second and seventh, have a maximum number of close higher harmonics and exhibit thus maximal roughness. In this way Helmholtz was even able to calculate the degree of dissonance, and the results of his calculations agree very well with musical experience. Also other more subtle effects, e. g. that thirds in low tones are dissonant — well known to composers for a long time — follow easily from his theory.

The theory of harmony of Helmholtz was first hailed enthusiastically and then criticized severely. When Schönberg was writing his famous *Harmonielehre*[21] criticism was on his height. But Schönberg was fully aware of the symbolic character of scientific theories[22]:

> In dieser Hinsicht ist es also für die Deutung harmonischer Probleme wenig von Belang, ob die Funktion der Obertöne von der Wissenschaft schon widerlegt ist oder erst angezweifelt wird. Gelänge es ... die Probleme sinnvoll zu deuten und übersichtlich darzustellen, ... so könnte doch der Zweck erreicht werden, auch wenn sich, was ja nicht unbedingt sein muss, nach einiger Zeit herausstellte, dass beide, Obertontheorie und Deutung, falsch sind.

And he continues:

> Und so werde ich denn bei meinen Betrachtungen von der vielleicht unsicheren Obertonlehre ausgehen, weil, was ich von ihr ableiten kann, sich zu decken scheint mit der Entwicklung der harmonischen Mittel.

Today Helmholtz theory of harmony is generally accepted in the psychoacoustic literature, and modern modifications have little added to it. This can be seen from a modern calculation of the degree of dissonance[23] which agrees astonishingly well with the evaluation of Helmholtz made 100 years earlier.

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21. Arnold SCHÖNBERG, *Harmonielehre*, Wien: Universal, 1911/1922.
22. l. c., p. 15 f.
23. See R. Plomb, l. c., p. 70 f.

Let me shortly recapitulate: by Muller's law of the specific energy of the senses Helmholtz was led to his concept of signs which saw the immense separation between the sensations and the outside world. He also layed emphasis on the lawful relations between the signs, most suitably expressed by mathematical equations. Hertz adopted this aspects in his concept of sign or symbol, but he stressed that these symbols are creations of our mind and that even in a well defined scientific context the symbols are by no means unique. As the changes between the third and fourth edition of the *Lehre von den Tonempfindungen* show, Helmholtz was aware that his original concept of sign may have been too realistic. This had been clearly recognized by the extremely sensitive Ernst Mach. When he criticizes Helmholtz' theory of harmony, based on the coincidence of higher harmonics he states: This coincidence of the harmonics exists only for the analyzing mind and has nothing to do with sensation and he stresses the ... *symbolic* character of Helmholtz' theory of harmony[24].

I come now to the second part, which investigates the concept of sign, phantom or symbol *(Scheinbilder oder Symbole)* in the work of Hertz. Hertz was the most favourite pupil of Helmholtz, and Helmholtz writes of him[25]:

> Durch seltenste Gaben des Geistes und des Charakters begünstigt, hat er in seinem leider so kurzen Leben eine Fülle fast un verhoffter Früchte geerntet . . . — In alter, klassischer Zeit würde man gesagt haben, er sei dem Neide der Götter zum Opfer gefallen. . . . Es war ein Geist, der ebenso der höchsten Schärfe und Klarheit des logischen Denkens fähig war, wie der grössten Aufmerksamkeit in der Beobachtung unscheinbarer Phänomene.

He is principally known for his discovery of radio-waves, a discovery which turned out to be not only of the utmost importance for our daily live, but it also was the final prove that there is no action at a distance in electrodynamics and hence Maxwell's theory of electricity and magnetism was the correct one. His discovery, which demanded a lot of experimental ingenuity, was by now means the result of a happy accident — as e. g. the discovery of the X-rays by Röntgen or radioactivity by Becquerel, but the

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24. Dr. E. MACH, *Analyse der Empfindungen*, 9. ed., Jena: Fischer, 1922, p. 236 f.

25. In the foreword of Heinrich HERTZ, *Die Prinzipien der Mechanik, in neuem Zusammenhange dargestellt*, Leipzig: Barth, 1894, p. VII f.

fruit of his deep understanding of Maxwell's theory. So one can say that it was only possible for a scientist who united the properties quoted by Helmholtz to achieve such a discovery.

Hertz develops his concepts of sign and symbol in his work *Die Prinzipien der Mechanik, in neuem Zusammenhange dargestellt*, and before I come to the aim and content of this work, I shall quote longer pieces from the introduction, which are also quoted by Cassirer in the first volume of his *Philosophie der symbolischen Formen*[26] where he calls them the most poignant expression of the application of the epistomology of symbolic forms in science. Hertz writes (l. c., p. 1 f.):

> Es ist die nächste und in gewissem Sinne wichtigste Aufgabe unserer bewussten Naturerkenntnis, dass sie uns befähige, zukünftige Erfahrung vorauszusehen, um nach dieser Voraussicht unser gegenwärtiges Handeln einrichten zu können. . . . Das Verfahren aber, dessen wir uns . . . bedienen ist dieses: Wir machen uns innere Scheinbilder oder Symbole der äusseren Gegenstände, und zwar machen wir sie von solcher Art, dass die denknotwendigen Folgen der Bilder stets wieder Bilder seien von den naturnotwendigen Folgen der abgebildeten Gegenstände. . . . — Die Bilder, von welchen wir reden, sind unsere Vorstellungen von den Dingen; sie haben mit den Dingen die eine wesentliche Übereinstimmung, welche in der Erfüllung der genannten Forderung liegt, aber es ist für ihren Zweck nicht nötig, dass sie irgend eine weitere Übereinstimmung mit den Dingen hätten.

Before I proceed further, I want to stress the difference between the *Zeichen* (signs) of Helmholtz and the *Bilder* or *Symbole* (pictures, symbols) of Hertz. For Helmholtz' signs are related to the sensual impressions, the symbols of Hertz are creations of the mind! Common to both concepts is the separation from the external things and common is also the emphasis layed on the lawful relations.

Since the pictures are our creations, they are not unique[27]:

> Eindeutig sind die Bilder . . . noch nicht betimmt durch die Forderung, dass die Folgen der Bilder wieder Bilder der Folgen seien. Verschiedene Bilder derselben Gegenstände sind möglich und diese Bilder können sich nach verschiedenen Richtungen unterscheiden.

26. l. c., p. 5.
27. l. c., p. 2.

Apart from the stringent conditions of logical and empirical correctness there are the less stringent ones of utility and clarity and these are the points of view according to which ones has to judge the value of physical theories and their representation. In his *Prinzipien der Mechanik* Hertz develops a new set of pictures which is constructed in order to describe (classical) mechanics. He first describes the time-honoured picture going back to Newton and D'Alembert. It starts with three concepts : Space, Time, Mass and Force. The connection between these concepts is given by the famous three laws of Newton[28]:

> Lex I : Corpus omne perservare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.
>
> Lex II : Mutationem motus proportionalem esse vi motrici impressæ, & fieri secundum lineam rectam qua vis illa imprimitur.
>
> Lex III : Actioni contrariam semper & æqualem esse reactionem: sive corporum duorum actiones in se mutuo semper esse æquales & in partes contrarias dirigi.

The concept of force as cause of the true mouvement is essential for Newton. He writes[29]:

> Motus autem veros ex eorum causis, effectibus, & apparentibus differentiis colligere ; & contra ex motibus seu veris seu apparentibus eorum causa & effectus, docebitur fustus in sequentibus. Hunc enim in finem Tractatum sequentem composui.

But in the most important contribution to mechanics after Newton's Principia, in the Traité de Dynamique of d'Alembert[30], the emphasis has shifted completely. In the introduction to his treatise he writes[31]:

> [...] ainsi on ne sera point surpris, que [...] j'aie pour ainsi dire, détourné la vue de dessus les *causes motrices*, pour n'envisager uniquement le Mouvement qu'elles produisent ; que j'aie entièrement proscrit les forces inhérentes au Corps en Mouvement, êtres obscurs et métaphysiques, qui ne sont capables que de répandre les ténèbres sur une Science claire par elle-même.

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28 Isaac NEWTON, *Philosophiœ Naturalis Principia Mathematica*, Ed. ult. Amsterdam : 1723, p. 12 f.

29. l. c., p. 11.

30. D'ALEMBERT, *Traité de Dynamique*, 2. ed., Paris : Fuchs, 1796.

31. l. c., p. xv.

And yet, Newton and D'Alembert both treat the same subject, and what is even more astonishing, they come to the same results.

Hertz' « Mechanics » goes in the same direction as the *Traité* of d'Alembert. He wants to eliminate the concept of force from mechanics and bases it on the concept of Space, Time and Mass alone. The price he has to pay is that he has in addition to the visible masses to invent other invisible masses obeying the same laws as the visible ones. But having done this he could show that all mouvements observed in nature can be described in his way without recurrence to the concept of force but with the same results for the motion as Newtonian mechanics.

I shall not dwell on the admirable consequent construction of Hertz, since it turned out to be not decisive for the future development. One of his main objectives, the elimination of an action at the distance, has been reached by the theory of general relativity and as the general frame a picture one based on time, space and energy, governed by Hamilton's principle of least action is nowadays assumed to be the most relevant one. But before I come back to different pictures of mechanics, I shall elaborate on some remarks also made by Hertz in a conference given in Heidelberg 1889[32]:

> Man kennt seine <Maxwells> im Jahre 1865 veröffentlichte Arbeit unter dem Namen elektromagnetische Lichttheorie. Man kann diese wunderbare Theorie nicht studieren, ohne bisweilen die Empfindung zu haben, als wohne den mathematischen Formeln selbständiges Leben und eigener Verstand inne, als seien dieselben klüger als wir, klüger sogar als ihr Erfinder, als gaben sie uns mehr heraus, als seinerseit in sie hineingelegt wurde.

Also Cassirer is fascinated by this feature and he writes[33] that this proper meaning and this detecting force *(Eigenbedeutung und Spürkraft)* of the *formulæ* is one of the most attractive problems of epistemology of science.

As an illustration I shall give a collection of the mathematical relations in different symbolic pictures of the same mechanical problem, the planetary motion around the sum.

Let us take as starting point the famous laws derived by Kepler from the observations of Tycho Brahe. They are, in the formulation of Isaac Newton[34]:

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32. H. HERTZ, *Ges. Werke I*, Leipzig: Barth, 1894, p. 344.
33. E. Cassirer, *Determinismus*, p. 173.
34. 1. c., p. 375, 360.

> I, II. Planetæ moventur in Ellipsibus umbicilicum habentibus in centro Solis, & radiis ad centrum illud ductis areas describunt temporibus proportionales.
>
> III. Planetarum ... circa Solem tempora periodica esse in rationem sesqiplicata mediocrium distantiarum à Sole.

Newton could show in his *Principia* that these laws follow (with some essential modifications, not yet accesible to observation in his time) from his laws, if he made the important and in he scientific community of that time extremely controversial proposition on the « gravitational force »[35]:

> Propositio II, Theorema II : Vires, quibus Planetæ perpetuo retrahantur a motibus rectilineis, & in Orbibus suis retinentur, respicere Solem & esse reciproce ut quadrata distantiarum ab ipsius centro.

In modern notation (going back to Leibniz) we can express this through the differential equation :

$$\frac{d\, m\, \dot{\vec{x}}}{dt} = \kappa \frac{\vec{x}}{|\vec{x}\,|^3} \,.$$

The right-hand side of this equation is the gravitational force, and the solutions of this equation are conic sections of which the Kepler ellipses are special cases.

In a picture developed by Euler and Lagrange the orbits of motion are those which minimalize a certain quantity, the so called action. This action is the time integral over a quantity now governing « the dynamics » of the system, the Lagrange function $L(\vec{x}, \dot{\vec{x}})$. For the problem of the motion of a planet around the sun the Lagrangian function is :

$$L(\vec{x}, \dot{\vec{x}}\,) = \frac{m}{2}\dot{\vec{x}}^{\,2} - \frac{\kappa m}{|\vec{x}\,|} \,.$$

The condition of a minimal action reflects itself in the Euler-Lagrange equations :

$$\frac{d}{dt}\frac{\partial L(\vec{x}, \dot{\vec{x}})}{\partial \dot{x}_j} = \frac{\partial L(\vec{x}, \dot{\vec{x}})}{\partial x_j} \,.$$

35. l. c., p. 362.

It is a trivial exercise to show that the Lagrange function given above inserted in the Euler-Lagrange equations just gives the differential equation equivalent to Newtons laws and proposition.

The approach to mechanics has been further formalized by Hamilton, Jacobi and Poisson. Here the fundamental «dynamical quantity[36]» is the Hamilton function, which depends on the position and momentum coordinates. For our case under consideration is just the total energy:

$$H = \frac{\vec{p}^{\,2}}{2m} + \frac{\kappa m}{|\vec{x}\,|} \,.$$

The equations of motion can be expressed by the Poisson brackets

$$\dot{x}_j = \{x_j, H\} \qquad \dot{p}_j = \{p_j, H\} \,,$$

with

$$\{u, v\} = \sum_{k=1}^{3} \left(\frac{\partial u}{\partial x_k} \frac{\partial v}{\partial p_k} - \frac{\partial v}{\partial x_k} \frac{\partial u}{\partial p_k} \right) \,.$$

For the position $\vec{x}$ and the momentum $\vec{p}$ the Poisson brackets give:

$$\{x_k, p_j\} = \delta_{kj} \qquad \{x_k, x_j\} = \{p_k, p_j\} = 0 \,.$$

Though this different pictures of describing a simple physical problem look rather different, they lead at the end to the same differential equation. At the end one has to solve this equation and compare it with the data.

The reason for the «formal development» was manifold: d'Alembert could with his method, looking at mechanics «plutôt comme la Science des effets, que comme celle des causes» solve problems which cannot be treated by Newtonian mechanics in its original form. Some of the newer formal development was done in order to give new tools to treat the mathematically extremely

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36. We see that the expression «dynamical» has lost his original sense, since we do not use forces at all; this problem has already been encountered by d'Alembert, who states «Ce nom (i. e. Dynamique) [...] pourroit paroître [...] ne pas convenir à ce Livre [...]», l. c., p. xxviij.

complicated problems of celestial mechanics. But I am sure that the beauty and rich mathematical structure of the formalisms were also an extremely important motivation. The structure of the Hamilton-Jacobi formalism with Poisson brackets turned out to be the road to new developments, and indeed, the inherent intelligence and detecting power of the formalism become clear if we come to quantum mechanics, where we want not to describe the planetary motion around the sun, but e. g. a system proton-electron. In that case the symbols have changed their meaning completely, it is even inadmissible to speak of orbits of the electron, the position and momentum coordinates x p are now operators, but the structure of the relations is remarkably similar. With the space and momentum coordinates the Hamilton function of classical mechanics becomes also an operator, which has, however, exactly the same relation to space and momentum observables as in classical physics :

$$H = \frac{\vec{p}^2}{2m} + \frac{\kappa m}{|\vec{x}\,|} .$$

The equations describing the «dynamical» evolution are structurally similar to the classical ones :

$$\dot{x_j} = [x_j, H] \qquad \dot{p_j} = [p_j, H] ,$$

but here the Poisson brackets are replaced by the (much simpler) commutator :

$$[u, v] = u\,v - v\,u$$

The structural relation between space and momentum observables is again very similar to the classical relations :

$$[x_k, p_j] = i\delta_{kj} \qquad [x_k, x_j] = [p_k, p_j] = 0$$

Let me try to summarize a complicated history in a few words : The symbolic description of nature was already present in real birth of modern science through Newton. In making his proposition II on the gravitational force he introduced a concept which was a *σκάνδαλον* to more metaphysically inclined scientists and natural philosophers as Huygens and Leibniz. It seems that

Newton, as many true revolutionaries, was not satisfied with his « symbolic » introduction of the gravitational force and perhaps it was mostly his mathematical genius which led him to pursue this road. D'Alembert however was actively propagating science in this direction and I think that progress in science was largely made possible by this liberation from metaphysical ballast, the retarding moment of the German *Naturphilosophie* proves this *e negativo*. In Helmholtz' and Hertz' scientific work the symbolic character of science is made clear by their clear and specific — albeit not very elaborated — concepts of sign and symbol. I hope to have shown you in the example that the symbolic character of physics has become even more prominent in modern science and fits marvellously into Cassirer's concept of symbolic forms as a general scheme of all cultural activities.

Hans Günther DOSCH