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Ungelöste Probleme

Nr. 54: We discuss a problem about a set of symmetric functions of the zeroes of $J_\nu(z)$, the Bessel function of first kind. Let the positive zeroes of $z^{-\nu}\, J_\nu(z)$ be denoted by $j_{\nu,m}$, $m = 1, 2, \ldots$.

Let

$$\sigma_{2n} = \sum_{m=1}^{\infty} (j_{\nu,m})^{-2n}, \quad n = 1, 2, \ldots. \tag{1}$$

It was proved by Nand Kishore [2] that

$$\sigma_{2n}\left(\frac{1}{2}\right) = (-1)^{n-1} \frac{2^{2n-1}}{(2n)!} B_{2n} \tag{2}$$

$$\sigma_{2n}\left(-\frac{1}{2}\right) = (-1)^{n} \frac{2^{2n-2}}{(2n)!} G_{2n} \tag{3}$$

where

$$B^n = (B+1)^n, \quad n \neq 1$$

and

$$G_n = 2\,(1-2^n)\, B_n .$$

B_n and G_n are the well known Bernoulli's and Genocchi numbers respectively. The properties of $\sigma_{2n}(0)$ were discussed by Carlitz [1].

Letting

$$S_{2n}(\nu) = \sum_{m=1}^{\infty} (-1)^{m+1} (j_{\nu,m})^{-2n} \tag{4}$$

$$K_{2n+1}(\nu) = \sum_{m=1}^{\infty} (-1)^{m+1} (j_{\nu,m})^{-(2n+1)} \tag{5}$$

It is easy to prove that

$$S_{2n}\left(\frac{1}{2}\right) = \frac{(-1)^{n-1}\,(2^{2n-1}-1)\, B_{2n}}{(2\,n)!} \tag{6}$$

and

$$K_{2n+1}\left(-\frac{1}{2}\right) = \frac{(-1)^n\, E_{2n}}{4\cdot(2\,n)!} \tag{7}$$

where E's are the Euler's numbers which are defined by the symbolic relation $(E+1)^k + (E-1)^k = 0$.

The problem is – Find a generating function for $S_{2n}(\nu)$ and $K_{2n+1}(\nu)$.

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REFERENCES

[1] L. Carlitz, *A Sequence of Integers Related to Bessel Functions*, Proc. Amer. Math. Soc. *14*, 1–9 (1963).

[2] Nand Kishore, *The Rayleigh Function*, Proc. Amer. Math. Soc. *14*, 527–533 (1963).