Subject Matter.

Objekttyp: **Chapter**

Zeitschrift: **L'Enseignement Mathématique**

Band (Jahr): **5 (1959)**

Heft 1: **L'ENSEIGNEMENT MATHÉMATIQUE**

PDF erstellt am: **21.07.2024**

THE SUBJECT MATTER.

Mathematicians are prone to think of mathematics and school arithmetic as two separate and only slightly related disciplines. Thus school arithmetic, or Rechnung, or computation is a set of rules and mechanical operations to be learned in the early years of schooling, while arithmetic, related to Algebra is the theory of numbers and is the beginning of the study of mathematics. This point of view was present in many of the reports, but not all. The subject matter thus studied in these years can be classified as Number or Computation, Algebra, Geometry, and Numerical Trigonometry.

Under Number the following topics are universally studied. Numbers as counting numbers, first to five, then ten, then 20, then 100, then 1000, and up to 10,000,000. The decimal system of notation; the Roman numerals to thousand. The addition and subtraction of whole numbers with and without carrying or borrowing. In many countries the Austrian Method [2]) of subtraction is mandatory right from the start.

Multiplication tables are memorized and include all combinations up to 10×10, some countries demanding up to 12×12, and one country including all combinations to 20×20. The multiplication of whole numbers begins with a single digit multiplier, and then two and three digit multipliers, preceded by multiplication by powers of 10. Division, considered the most difficult process is the last to be taught beginning with divisors of 2, 3, 5 and 10; and finally 2 and 3 digit divisors. Once the process has been taught there are applications to simple problems of everyday life.

The meaning of simple fractions as equal parts of a whole such as 1/2, 1/3, or 1/5 is introduced early, but actual operations with fractions (called common in America and often referred to as vulgar elsewhere) is delayed until all operations with the whole numbers have been taught. The first operation consists

---

2) The Austrian method uses the principle of adding the same number to the minuend and subtrahend. Thus in 42-18, ten is added to the 2 in 42 to give 40 and 12, and ten is also added to the one ten in 18 to give 28. Then the substracter says " 8 and what gives 12 ? " Answer " 4 ". Then " 2 and what gives 4 ? " Answer " 2 ". The difference is 24.

of changing (so called reducing) of fractions to the same denominator, and in quite a few countries the method is based on that of finding a Least Common Multiple. Then the four operations with fractions are taught, including operations with mixed numbers. Finding the Greatest Common Factor is also stressed in many countries.

Decimal fractions, or better fractions written in decimal notation are usually delayed until the 5th or 6th year of instruction. Before this, however, countries using the metric system or a decimal money system have included numbers written with a decimal point (or comma). However the four operations on mixed decimals, that is, numbers with whole and fractional parts written in decimal notation, is taught in all countries. The equivalence of common fractions to decimal fractions and making the transformation from one form to the other culminates this aspect of computation.

Along with the whole numbers, systems of measures are introduced and gradually extended so that operations can be performed with them. The measures include the local national ones, as well as the metric system, but the latter is delayed until the 7th of 8th school year in all countries that do not use it as their basic system. The measures include those for length, area, volume, capacity, weight, money and time. Exchange of money is taught in most countries.

Percentage, as a special topic is taught in all countries. Much emphasis is put on this topic and applications are made to simple and compound interest, discount, chain discounts, profit and loss, commission, borrowing, instalment purchases, stocks and bonds, and other business affairs. The application of arithmetic to business and daily life problems seems to have grown in amount and stress in all countries during the past few decades.

Ratio and proportion are taught universally, a very common part of this work being the rule of three. A few countries still teach alligation simple and alligation compound [3] with

[3] Alligation refers to the rules of finding the proportions of various ingredients in a mixture to meet specified requirements on cost or strength. The word disappeared from American text books more than 40 years ago.

applications to mixtures, work problems, and the like. There are also applications to scale drawing and map-reading. The unitary method was commonly mentioned as required work.

All countries teach the concept of average, whereby is meant finding the arithmetic mean of set of numbers. A few countries teaching divisibility by 2, 3, 5, and 9, are using divisibility by 9 as a check on the operations of multiplication and division. Prime numbers receive scant or no attention. It is noteworthy that no country reported teaching numeration to any other base than 10, nor did any country report teaching the fundamental laws of arithmetic (as a Ring) in developing the rules of operation. Thus it must be generally concluded that the teaching of arithmetic is looked upon as the development of a tool with many tricks and manipulations, that are to be used in special types of practical problems. Theoretical considerations are at a minimum. Reasoning is not demanded, but skill in manipulating is of the essence.

The work in Geometry begins with the study of the ruler, using it to mark off and to measure distances. The common geometric figures, square, rectangle, triangle and circle are illustrated by drawings and observed in nature. In all countries the first concepts taught are that of perimeter and area of the square and rectangle. The first approach to area is through the use of squared paper, where the squares in the interior of the figure are counted. The continuation of the study of geometry may be described as a gradual approach to demonstrative methods by way of concrete measurements, intuitive study of properties of plane figures and solids, simple deductions, and finally a study of the nature of proof. In the intuitive approach there is covered the measurement of lines and angles, the perimeter and area of plane figures including the circle, the areas and volumes of the common solids — cube, rectangular prism, regular prisms, right circular cylinder, regular pyramid, right circular cone, and the sphere. The results are determined by practical means and the use of models, and then generalized by formulas. The work on geometry next concerns itself with parallel lines, perpendicular lines, angle bisectors, congruent triangles and rectangles and is accompanied by much construction work with compasses and rulers, as well as set-squares, triangles and protractors. Many of the usual theorems of plane geometry are thus evolved as facts or relation without any formal deductive system. The third aspect of intuitive geometry may be called relational, and here there is studied the base angles of an isosceles triangle, complementary, supplementary and vertical angles, the relation of sizes of angles to the opposite side of a triangle, the sum of the angles of a triangle and the Pythagorean relationship for a right triangle. At this place, in instruction practically all countries report the finding of squares and square roots of numbers, both by numerical methods and by the use of tables. The use of Newton's method of division and averaging the quotient and divisor is the only method reported, the Euclidean method having gone into discard at this stage of learning.

Along with the Pythagorean theorem, ratio and proportion are introduced into geometry via similar triangles and then the sine, cosine and tangent ratios are defined for angles between 0° and 90°. These ratios are used to find sides and angles of right triangles from given numerical values for the other sides and angles. Applications are made of the trigonometric ratios and the Pythagorean theorem to practical problems, especially those of simple surveying and navigation. In this study such instruments as the clinometer, hypsometer, transit, sextant, and angle mirror are used. The work is accompanied by drawing to scale and the making of simple plans.

In most countries, but not all, by the age of 14 years or in grades 8 and 9, the children have been introduced to the axioms, the use of definitions, the simple syllogism and a deductive chain of theorems, that is, the study of Euclidean deductive plane geometry. The amount of deductive geometry taught by age 15 years (grade 9) varies from a study of only congruence and parallelism to that of completion of all theorems on rectilinear figures, the circle and angle measurement, similarity area, and the regular polygons. There is far more variation in the amount of geometry studied in the various countries, than there is in the content of algebra.

The approach to algebra is usually through the generalization of arithmetic by the use of letters for numbers. Thus in many countries arithmetic means introductory or manipulative algebra. Letters are used with fundamental operations and then equations are introduced. Other countries approach the simple equation through the study of formulas, where letters are used to formulate general rules. A third approach is to begin algebra by the study of positive and negative numbers, develop the laws of these numbers and formulate them with letters leading to identities such as

$$x + y = y + x$$

and

$$a\,(b + c) = ab + ac\,.$$

The stress in the algebra study at age 14 or 15 is on the solution of equations, first the simple equation in one unknown, then two equations in two unknowns (or three unknowns), and finally the quadratic equation. In so far as the reports show, the emphasis is on tricks and formulas and not on proof. The word theorem or proposition is a rarity in the first study of algebra. The remaining study of algebra is given over to special products and factoring; the linear function; direct and inverse variation; and the graph of the linear and quadratic function. Only two countries report more study including such topics as Highest Common Factor, Lowest Common Multiple, involution and evolution; fractional indices; surds; the function $y = \sqrt{x}$; and the derivative of a polynomial.

This, then is the picture of what the pupil has been taught. What does he really know ? This is hard to tell, but it can be said that the 15 year old in all countries, who has continued his study of mathematics through the first 9 or 10 years of school can compute in a mature manner with the positive rational numbers, in a decimal system of notation, even though he cannot rationalize what he does; he has a fairly useful and practical knowledge of geometry with respect to mensuration and common relationships; and he can manipulate algebraic expressions and solve equations and problems in a structureless system of algebra. He can make simple deductions, but his entire concept of proof, if any, is limited to that of theorems in geometry. He really does not know what mathematics is, or how it is applied, but he has a large body of information, upon which, if he is inclined or interested, a study of mathematics can be built in the ages 16 years to 21 years. The whole program, the world over, is overloaded with " doing " and it would appear that a reformation of this program with emphasis on " reasoning " and an elimination of much useless and extraneous busy work could enliven the subject and leave the 15 year old with a much clearer and stronger picture of what mathematics study really is.

School Organization and Time Allotted to Mathematics Study.

One thing is certain, and that is that the school organization and selection of students is unique in each of the countries having compulsory (and free) education. The starting age for grade 1, or first year of formal schooling varies from 5 years to 7 years of age. Thus by the age of 15 years, youth in the several countries have had from 8 to 10 years of schooling. But the number of years of instruction is modified somewhat by the fact that the number of clock-hours of instruction per week, and the number of weeks per year vary greatly from country to country. Thus the total allotted time for mathematics instruction compared with the entire instructional time in a given country varies from 20% to 9%. Statistics are boring, so here follow a few sample programs and time allottments that are indicative of the highest, lowest, and median of the countries reporting.

In all countries, the first four years of instruction is given in an elementary or folk-school, which all children regardless of ability or social origin may attend. The classes are taught by a teacher who teaches all subjects, that is, there is no special teacher for mathematics or the other branches of learning. The only exception to this statement is the use of special teachers