As in the case n even, it will suffice to take $r \equiv 3 \pmod 4$. Let 2^{e_r} denote the highest power of 2 dividing $r + 1$, and consider 2^{-e_r}T. For $e_r \leqslant 2$, we get the exponent -1; for $e_r \geqslant 3$ it follows that $\frac{1}{2}(r-1) \geqslant e_r$. Hence if $k + 2 < e_r$, we get the exponent $k + 2 - e_r$, while if $k + 2 \geqslant e_r$ we again get -1 (at most). Consequently

$$\Delta_n \equiv 0 \pmod{2^{3n-e+k+2}} \quad (k + 2 < e_r) \ , \tag{16}$$

$$\Delta_n \equiv 0 \pmod{2^{3n-1}} \quad (k + 2 \geqslant e_r) \ . \tag{17}$$

Comparing (16) and (17) with (13) and (14) we may accordingly state the following

THEOREM. *Let* $2^e \leqslant 2n < 2^{e+1}$ *and let* 2^k *denote the highest power of 2 dividing* n *or* n $-$ *1 according as* n *is even or odd. Then* Δ_n *as defined by (1) and (3) satisfies (16) and (17).*

Applying the Staudt-Clausen theorem, it follows from (3) that

$$p\Delta_n \equiv -\sum_{r>0} (-1)^{n-rm} \binom{n}{rm} \pmod p \ ,$$

where $p = 2m + 1$ is an odd prime. It can be shown that

$$\sum_{r>0} (-1)^{n-rm} \binom{n}{rm} \equiv \begin{cases} 2 & (p-1 \mid n) \\ \binom{m}{k} & (p-1 \mid n+k,\ 0 < k \leqslant m) \\ 0 & \text{otherwise.} \end{cases}$$

In particular the prime factors of the denominator of Δ_n are simple and cannot exceed $2n + 1$.

In connection with formula (4) above it may be of interest to cite the formula [2, p. 189]

$$(-1)^n \sum_{r=0}^{n} \binom{n}{r} \frac{B_{n+r+1}}{n+r+1} + \frac{n!\,n!}{2\,(2n+1)!} = 0 \ .$$

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