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Theorem (Krull). Let A be a local noetherian ring, E a finite module over A . Then:

- (i) The Krull topology of E is separated.
- (ii) Every submodule F of E is closed in E .
- (iii) The topology induced by E in a submodule F is the Krull topology of F .

Proof. (i) Let $F = \bigcap_{k \geq 0} m^k E = \overline{\{0\}}$. Then

$$mF = m((m^n E) \cap F) = (m^{n+1} E) \cap F = F$$

by the Artin-Rees lemma. Hence Nakayama's lemma implies that $F = \{0\}$.

(ii) Let $f: E \rightarrow E/F$ be the natural map. Then

$$f(\bar{F}) \subset f(F + m^k E) = f(m^k E) = m^k (E/F).$$

Hence $f(\bar{F}) \subset \bigcap m^k (E/F) = \{0\}$, using (i). But $f(\bar{F}) \subset \{0\}$ is equivalent to $\bar{F} \subset F$.

(iii) It is clear that $m^k F \subset (m^k E) \cap F$. Hence the Krull topology of F is finer than that induced by E ; in other words the inclusion $F \rightarrow E$ is continuous. Conversely the Artin-Rees lemma shows that

$$(m^{n+k} E) \cap F = m^k ((m^n E) \cap F) \subset m^k F$$

which proves that the induced topology is finer than the Krull topology of F .

REFERENCES

- [1] BOURBAKI, N. *Algèbre commutative*, ch. 3, 4. Hermann, Paris 1961.
- [2] GRAUERT, H. Ein Theorem der analytischen Garbentheorie und die Modulräume komplexer Strukturen, publications mathématiques, *I.H.E.S.*, n° 5, Paris 1960.
- [3] ——— *Lectures at Otaniemi* Finland 1967 (in this volume).
- [4] GROTHENDIECK, A. *Exposés 7-17 in Séminaire Cartan* Paris 1960/61.
- [5] GUNNING, R. C. and H. ROSSI, *Analytic functions of several complex variables*. Prentice Hall, Englewood Cliffs, N.J. 1965.
- [6] HOUZEL, C. *Exposés 18-21 in Séminaire Cartan*, Paris 1960/61.
- [7] MATHER, J. N. *Stability of C^∞ mappings II* (to be published in *Annals of Math.*).
- [8] NAGATA, M. Local rings. *Interscience*, New-York 1962.
- [9] NARASIMHAN, R. Introduction to the theory of analytic spaces. *Lecture notes in Mathematics*, n° 25. Springer, Berlin 1966.