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Autor: Stein, K.

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Proposition 5. If $f: X \underset{k}{\to} Y$ is continuous and Y is a Hausdorff space, G_f is closed in $X \times Y$.

Definition 2. A correspondence f is *proper* if f and f^{-1} are continuous.[1]

Proposition 6. If $f: X \underset{k}{\to} Y$, $f_1: X_1 \underset{k}{\to} Y_1$, $g: Y \underset{k}{\to} Z$ are proper, then $f \times f_1$ and $g \circ f$ are proper.

The junction of two proper correspondences need not, however, be proper. The diagonal mapping (I_X, I_X) serves as an example if X is not a Hausdorff space. If X is Hausdorff, the junction (f, f') of proper correspondences $f: X \underset{k}{\to} Y$ and $f': X \underset{k}{\to} Y'$ remains proper.

Proposition 7. Let $f: X \underset{k}{\to} Y$, $f_1: X \underset{k}{\to} Y_1$, $g: Y \underset{k}{\to} Z$ be continuous where all the spaces are locally compact. Then we have:

1) If f is proper, then (f, f_1) and (f_1, f) are proper,

2) If $g \circ f$ is proper and g^{-1} surjective, then f is proper,

3) If $g \circ f$ is proper and f surjective, then g is proper.

2. Holomorphic correspondences

We consider reduced complex spaces (X, θ) where X is assumed Hausdorff and where the structure sheaf θ has no nilpotent elements. For the definition and related concepts we refer to [8]. The structure sheaf is usually omitted in the notation.

Definition 3. Let X and Y be complex spaces. A correspondence $f: X \underset{k}{\to} Y$ is called *holomorphic* if

1) f is continuous,

2) the graph G_f is an analytic set in $X \times Y$.

If only the condition 2) is fulfilled, f is said to be *weakly holomorphic.*

Let $f: X \underset{k}{\to} Y$ be weakly holomorphic. Then f^{-1} is weakly holomorphic; furthermore, if $A \subset X$ is analytic in X, $f \mid A$ is weakly holomorphic. Since $\check{f}^{-1}(x) = G_f \cap (\{x\} \times Y)$, $x \in X$, is analytic in G_f, $f(x) = \hat{f}\left(\check{f}^{-1}(x)\right)$

1) Compare [3] where another notion of proper correspondence is defined.

is analytic in Y. If f is holomorphic and $A' \subset Y$ analytic in Y, then, since $\hat{f}^{-1}(A')$ is analytic in G_f and $\check{f}$ is proper, $f^{-1}(A') = \check{f}(\hat{f}^{-1}(A'))$ is analytic in X by Remmert's mapping theorem [11] (see also [8], p. 129).

The correspondences $f \times f_1$, (f, f'_1), and $g \circ f$ are holomorphic if the correspondences f, f_1, f'_1, and g are holomorphic.

A weakly holomorphic correspondence $f: X \underset{k}{\rightarrow} Y$ is called *reducible* resp. *irreducible* if G_f is reducible resp. irreductible. G_f is always a union of irreducible components $G^{(i)}$; let $f_i : X \underset{k}{\rightarrow} Y$ be the (weakly holomorphic) correspondence whose graph is $G^{(i)}$. Then the correspondences f_i are called the irreducible components of f and we write $f = \cup f_i$.

3. Meromorphic mappings

Let $f: X \underset{k}{\rightarrow} Y$ be a correspondence where X is a topological space A point $x \in X$ is called a *distinguished point of f* if there is a neighborhood U of x such that the restriction $f \mid U$ is a mapping (in the usual sense).

Definition 4. A holomorphic correspondence $f: X \underset{k}{\rightarrow} Y$ is called a *meromorphic mapping* if the following holds. If X is irreducible, then

1) f is irreducible,

2) There exists a distinguished point $x_0 \in X$ of f.

In the general case, if $X = \cup X^{(i)}$ is the decomposition of X into irreducible components, then there exist holomorphic correspondences $f_i : X \underset{k}{\rightarrow} Y$ such that

1) $f_i \mid X^{(i)}$ is a meromorphic mapping and $f_i \mid X - X^{(i)}$ is empty,

2) $f = \cup f_i$.

A meromorphic mapping f is *bimeromorphic* if f^{-1} is meromorphic.

We use the notation $f: X \underset{m}{\rightarrow} Y$ for a meromorphic mapping. Note that a meromorphic mapping is in general not a mapping in the strong sense.

An example of a meromorphic mapping is the correspondence f of $\mathbf{C}^2$ onto the extended complex plane $\mathbf{P}_1$ defined by $f(z_1, z_2) = \dfrac{z_1}{z_2}$ if $(z_1, z_2) \neq (0, 0)$, and $f(0, 0) = \mathbf{P}_1$.