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on A . Since $f \in H^0(\mathbf{P}^n, F^m)$, this gives the desired homogeneous polynomial.

To prove the theorem, it now suffices to consider all homogeneous polynomials which vanish on A without being identically zero and apply the Hilbert basis theorem.

5. MEROMORPHIC FORMS

Let X be a complex manifold. A holomorphic differential form is a form which in local coordinates can be written as a finite sum

$$\omega = \sum a_{i_1 \dots i_k} dz_{i_1} \wedge \dots \wedge dz_{i_k} \quad (5.1)$$

with holomorphic coefficients $a_{i_1 \dots i_k}$.

A form is called meromorphic if it has locally the form (5.1) with coefficients that are meromorphic functions. Every meromorphic function can be written locally as $f\omega$ where f is a meromorphic function and ω a holomorphic form. The exterior differentiation d , satisfying $d^2 = 0$, extends naturally to meromorphic forms.

Let D be a divisor of X and let $\Omega^p(k, D) = \Omega^p(X, k, D)$ be the sheaf of germs of meromorphic p -forms on X with poles only on D and of order $\leq k$, and let $\Omega^p = \Omega^p(X)$ be the sheaf of germs of holomorphic p -forms on X .

Lemma 5.1. There is a natural isomorphism

$$\Omega^p(k, D) \simeq \Omega^p \otimes \underline{F^k}.$$

Proof. A germ in $\Omega^p(k, D)$ at $a \in X$ is represented by a form $f\omega$, where f is a meromorphic function in a neighbourhood U of a , with poles only on D and of order $\leq k$, and ω is a holomorphic form on U . Now to f corresponds biuniquely a section $s \in \Gamma(U, F^k)$ (see Sect. 4), which gives a germ $s_a \in \underline{F^k}_a$. Also ω defines a germ $\omega_a \in \Omega^p_a$.

The desired mapping $\Omega^p(k, D) \rightarrow \Omega^p \otimes \underline{F^k}$ is now uniquely defined by

$$f\omega \rightarrow \omega_a \otimes s_a.$$

To see that it is an isomorphism, it is sufficient to observe that the inverse mapping of $\Omega^p \otimes \underline{F^k}$ into $\Omega^p(k, D)$ is induced by the bilinear mapping $\Omega^p \oplus \underline{F^k} \rightarrow \Omega^p(k, D)$, which is given by

$$(\omega_a, s_a) \rightarrow (f\omega)_a, \quad (a \in X).$$

where f is the meromorphic function determined by s_a by the procedure described just before Th. 4.1.

Now let X be a compact submanifold of \mathbf{P}^n and consider hyperplanes H_c in \mathbf{P}^n , given in homogeneous coordinates z_0, \dots, z_n by equations

$$\sum_0^n c_j z_j = 0 \text{ where } c = (c_0, \dots, c_n) \neq 0.$$

Theorem 5.2. There is an open dense set Ω in \mathbf{C}^{n+1} such that if $c = (c_0, \dots, c_n) \in \Omega$, the hyperplane section $D_c = H_c \cap X$ is a non-singular analytic subset of X .

The proof is omitted here.

Let $D = H \cap X$ be a non-singular hyperplane section of X . To D is then associated a positive line bundle F on X (see Sect. 4). By Kodaira's vanishing theorem there is a k_0 such that

$$H^q(X, \Omega^p \otimes \underline{F}^k) = 0, \quad (\forall q \geq 1, \forall k \geq k_0).$$

Using the isomorphism in Lemma 5.1, we have therefore proved.

Lemma 5.3. If D is a non-singular hyperplane section of a compact submanifold X of \mathbf{P}^n , then there exists k_0 such that

$$H^q(X, \Omega^p(k, D)) = 0, \quad (\forall q \geq 1, \forall k \geq k_0).$$

6. THE ATIYAH-HODGE THEOREM

We first recall two well-known theorems.

Let X be a paracompact C^∞ manifold and let \mathcal{E}^p be the sheaf of germs of C^∞ p -forms on X ($p=0, 1, \dots$).

Then the sequence

$$0 \rightarrow \mathbf{C} \xrightarrow{i} \mathcal{E}^0 \xrightarrow{d} \mathcal{E}^1 \xrightarrow{d} \mathcal{E}^2 \xrightarrow{d} \dots \quad (6.1)$$

is exact (Poincaré's lemma), and

$$H^q(X, \mathcal{E}^p) = 0, \quad (\forall q \geq 1, \forall p \geq 0), \quad (6.2)$$

because the \mathcal{E}^p are fine sheaves, i.e. they have partitions of unity. From (6.1) we get the sequence

$$0 \rightarrow \Gamma(X, \mathcal{E}^0) \rightarrow \Gamma(X, \mathcal{E}^1) \rightarrow \dots,$$

which need not be exact. Put