

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 14 (1968)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: COMPACT ANALYTICAL VARIETIES
Autor: Narasimhan, Raghavan

Bibliographie
DOI: <https://doi.org/10.5169/seals-42344>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 31.01.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Here $L(z-a)$ is the Levi form of p at the point a . Now, since p is strongly plurisubharmonic, we can choose the coordinates so that $L(z-a) = |z-a|^2$. Then we see that if ζ is an eigenvector corresponding to an eigenvalue < 0 of the symmetric matrix of the real quadratic form $\operatorname{Re} Q(z) + L(z)$, then $i\zeta$ is an eigenvector corresponding to an eigenvalue > 0 . Hence the number of negative eigenvalues is $\leq d$, since the real dimension of X is $2d$. Thus the index of the critical point a is $\leq d$.

Now using Lemma 7.3 (b), we see that

$$H_r(U_\beta, \mathbf{Z}) = 0, \quad (\forall r > d).$$

From this it follows that

$$H_r(X, \mathbf{Z}) = 0, \quad (\forall r > d),$$

because the singular cycles defining the homology groups $H_r(X, \mathbf{Z})$ have compact supports, and any compact subset of X is contained in some compact set K with a corresponding $U_\beta \supset K$.

A refinement of the above argument leads to the stronger (homotopy) statement:

Any Stein manifold of (complex) dimension d has the same homotopy type as a CW complex of (real) dimension $\leq d$. (See [6]).

Moreover, the Lefschetz theorem has an analogue in homology and in homotopy [6]. The latter, for example, asserts that, if V, D are as in Th. 7.1, then the relative homotopy groups $\pi_q(V, D) = 0$ for $q < d$.

Th. 7.2 has been generalised in various directions. It has a relative analogue (relative to a Runge domain). Further, Th. 7.2 remains true if X is any Stein space (with singularities) of complex dimension d , but the corresponding cohomology statement is proved only for some other coefficient groups [5, 7]. Note that in view of the use of Poincaré duality, this does not lead to a Lefschetz theorem for algebraic varieties with singularities.

REFERENCES

- [1] ANDREOTTI, A. and FRANKEL, Theodore, The Lefschetz theorem on hyperplane sections. *Ann. of Math.* 69 (1959), 713-717.
- [2] GRAUERT, H., Ueber Modifikationen und exzeptionelle analytische Mengen. *Math. Ann.* 146 (1962), 331-368.
- [3] GRAUERT, H., and REMMERT, R., Bilder und Urbilder analytischer Garben. *Ann. of Math.* 68 (1958), 393-443.
- [4] HÖRMANDER, L., *An introduction to complex analysis in several variables*. Van Nostrand, 1966.

- [5] KAUP, L., Eine topologische Eigenschaft Steinscher Räume, *Göttinger Nachrichten*, (1966) 213-224.
- [6] MILNOR, J., *Morse Theory*, Annals of math. Studies, no. 51.
- [7] NARASIMHAN, R., On the homology groups of Stein spaces, *Inventiones Math.* 2 (1967), 377-385.
- [8] DE RHAM, G., *Variétés différentiables*, Hermann, Paris, 1955.

(Reçu en mars 1968.)

Institut de Mathématiques
Université de Genève.