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Here $L(z-a)$ is the Levi form of p at the point a. Now, since p is strongly plurisubharmonic, we can choose the coordinates so that $L(z-a) = |z-a|^2$. Then we see that if ζ is an eigenvector corresponding to an eigenvalue < 0 of the symmetric matrix of the real quadratic form Re $Q(z) + L(z)$, then $i\zeta$ is an eigenvector corresponding to an eigenvalue > 0. Hence the number of negative eigenvalues is $\leqslant d$, since the real dimension of X is $2d$. Thus the index of the critical point a is $\leqslant d$.

Now using Lemma 7.3 (b), we see that

$$H_r(U_\beta, \mathbf{Z}) = 0\,, \qquad (\forall r > d)\,.$$

From this it follows that

$$H_r(X, \mathbf{Z}) = 0\,, \qquad (\forall r > d)\,,$$

because the singular cycles defining the homology groups $H_r(X, \mathbf{Z})$ have compact supports, and any compact subset of X is contained in some compact set K with a corresponding $U_\beta \supset K$.

A refinement of the above argument leads to the stronger (homotopy) statement:

Any Stein manifold of (complex) dimension d has the same homotopy type as a CW complex of (real) dimension $\leqslant d$. (See [6]).

Moreover, the Lefschetz theorem has an analogue in homology and in homotopy [6]. The latter, for example, asserts that, if V, D are as in Th. 7.1, then the relative homotopy groups $\pi_q(V, D) = 0$ for $q < d$.

Th. 7.2 has been generalised in various directions. It has a relative analogue (relative to a Runge domain). Further, Th. 7.2 remains true if X is any Stein space (with singularities) of complex dimension d, but the corresponding cohomology statement is proved only for some other coefficient groups [5, 7]. Note that in view of the use of Poincaré duality, this does not lead to a Lefschetz theorem for algebraic varieties with singularities.

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