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THE COHERENCE OF DIRECT IMAGES

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INTRODUCTION

The coherence of the direct images of coherent sheaves was treated in the paper [1]: H. Grauert: Ein Theorem der analytischen Garbentheorie und die Modulräume komplexer Strukturen (*Pub. Math.* IHES 1960, pp. 5-64, corrections 1963). This paper deals with the most general case and its technique is very difficult. The main point in the proof is the Hauptlemma on page 47. Here a proof of this Hauptlemma in the case of regular families of compact complex manifolds and locally free analytic sheaves is given. Although this special case is easier than the general, the ideas are practically the same. Therefore these lecture notes of some talks given by H. Grauert, Helsinki 1967, may lead to an understanding of the general proof. In these notes only the Hauptlemma is proved. The proof of coherence is omitted. This part is more formal and can be done like in [1] on p. 55. See [1] for applications of the theorem.

A detailed presentation of the proof in the general case is given also by Knorr [2].

COHOMOLOGY THEORY

In this paper we use Čech cohomology. We shall briefly show how this cohomology is defined. In the following discussion X denotes a connected complex analytic manifold, $\mathcal{O}$ is the sheaf of germs of holomorphic functions and S a sheaf of $\mathcal{O}$-modules. Let $\mathfrak{U} = \{ U_\iota \}_{\iota \in J}$ be an open covering of X. We put $U_{\iota_0 \dots \iota_l} = U_{\iota_0} \cap \dots \cap U_{\iota_l}$. We consider cochains of order l with values in S. Let us put $C^l(\mathfrak{U}, S) = \{ \xi \}$ where ξ denotes a full collection of crossections $\xi_{\iota_0 \dots \iota_l}$ over all $U_{\iota_0 \dots \iota_l}$. We always assume that $\xi_{\iota_0 \dots \iota_l}$ is anticommutative in its indices. In the system $\{ C^l(\mathfrak{U}, S) \}$ we have the