

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 14 (1968)
Heft: 1: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE COHERENCE OF DIRECT IMAGES
Autor: Grauert, H.
Kapitel: Existence of admissible refinements of measure coverings
DOI: <https://doi.org/10.5169/seals-42345>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 28.01.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

$= \{ \hat{U}_i(\rho) \}_1^{i^*}$ is a Stein covering of $X(\rho)$. We say then that $\hat{\mathfrak{U}}(\rho)$ is a measure covering of $X(\rho)$.

Admissible refinements of measure coverings. Let $\hat{\mathfrak{U}}(\rho)$ and $\hat{\mathfrak{U}}^*(\rho)$ be two measure coverings of $X(\rho)$. We say that $\hat{\mathfrak{U}}^*(\rho)$ is an admissible refinement of $\hat{\mathfrak{U}}(\rho)$ if the following conditions hold:

- 1) $U_i^* \subset \subset U_i$ for each i .
- 2) If $U_{i_0 \dots i_\lambda}^* = U_{i_0}^* \cap \dots \cap U_{i_\lambda}^*$ we put $(U_{i_0 \dots i_\lambda}^*)_v = \Phi_v^{-1}(U_{i_0 \dots i_\lambda}^* \times E^n(\rho))$ for each $v \in \{i_0 \dots i_\lambda\}$. It is now required that $(U_{i_0 \dots i_\lambda}^*)_v \subset (U_{i_0 \dots i_\lambda}^*)_\mu$ for all $v, \mu \in \{i_0 \dots i_\lambda\}$.
- 3) $\hat{U}_{i_0 \dots i_\lambda}^* = \hat{U}_{i_0}^* \cap \dots \cap \hat{U}_{i_\lambda}^* \subset (U_{i_0 \dots i_\lambda}^*)_\mu$ for each $\mu \in \{i_0 \dots i_\lambda\}$.

EXISTENCE OF ADMISSIBLE REFINEMENTS OF MEASURE COVERINGS

Existence Theorem. For every fixed integer s we can find, for some $\rho > 0$, a sequence $\mathfrak{U}_s \ll \mathfrak{U}_{s-1} \ll \dots \ll \mathfrak{U}_1 \ll \mathfrak{U}_0$ of finer measure coverings of $X(\rho)$ each of which is an admissible refinement of the following.

Proof. We first construct a measure covering of $X(\rho)$ for some $\rho < \min \rho_i$. Let $\mathfrak{U}_0 = \{ \mathfrak{U}_i \}_1^{i^*}$ be a Stein covering of X_0 such that $U_i \subset \subset W_i$ for $i \in \{1, \dots, i^*\}$. Choose a fixed $\rho_0 < \min \rho_i$. Now the open sets $\Phi_i^{-1}(U_i \times E^n(\rho_0))$ cover X_0 and hence they also cover $X(\rho)$ for some sufficiently small ρ . Hence \mathfrak{U}_0 defines a measure covering of $X(\rho)$. It is also clear that \mathfrak{U}_0 defines a measure covering of $X(\rho')$ for each $\rho' \leq \rho$. Let us now construct \mathfrak{U}_1 . We let $\mathfrak{U}^* = \{ U_i^* \}_1^{i^*}$ be a Stein covering such that $U_i^* \subset \subset U_i$ always holds. Now we can find $\rho_1 \leq \rho$ such that $\{ \hat{U}_i^*(\rho_1) = \Phi_i^{-1}(U_i^* \times E^n(\rho_1)) \}_1^{i^*}$ cover $X(\rho_1)$. Hence $\hat{\mathfrak{U}}^*(\rho_1)$ and $\hat{\mathfrak{U}}(\rho_1)$ are measure coverings of $X(\rho_1)$. But we do not yet know if $\hat{\mathfrak{U}}^*(\rho_1) \ll \hat{\mathfrak{U}}(\rho_1)$. We claim that if $\rho_2 \leq \rho_1$ is sufficiently small then $\hat{\mathfrak{U}}^*(\rho_2) \ll \hat{\mathfrak{U}}(\rho_2)$. For suppose this is false. Say that 2) fails for $\hat{\mathfrak{U}}^*(\rho_2)$ and $\hat{\mathfrak{U}}(\rho_2)$ when $0 < \rho_2 \leq \rho_1$. Hence $\Phi_v^{-1}(U_{i_0 \dots i_\lambda}^* \times E^n(\rho_2)) - \Phi_\mu^{-1}(U_{i_0 \dots i_\lambda} \times E^n(\rho_2))$ are non empty for suitable indices while $\rho_2 \rightarrow 0$. Choose a point x_t from each of these sets. Because $x_t \in X(\rho_1)$ which is relatively compact we may assume that $x_t \rightarrow x_0$. Obviously we get $x_0 \in \overline{U_{i_0 \dots i_\lambda}^*} - U_{i_0 \dots i_\lambda}$, a contradic-

tion because $\overline{U_{\iota_0 \dots \iota_\lambda}^*} \subset \overline{U_{\iota_0}^*} \cap \dots \cap \overline{U_{\iota_\lambda}^*} \subset U_{\iota_0 \dots \iota_\lambda}$. In the same way we can prove that condition 3) is satisfied if ρ_2 is sufficiently small and the theorem is clear.

GENERAL THEORY

Let G be an analytic manifold. We put $\hat{G} = G \times E^n(\rho_1)$ where ρ_1 is an n -tuple of positive numbers. Let $\pi: \hat{G} \rightarrow E^n(\rho_1)$ and $\mathfrak{P}: \hat{G} \rightarrow G$ be the projection maps. $\hat{G}^* \subset \hat{G}$ denotes an open subset and $G^* = \hat{G}^* \cap G \times \{0\}$.

The set G^* can be identified with an open subset of G . We denote by $\alpha: G^* \times E^n(\rho_1) \rightarrow \hat{G}^*$ a biholomorphic fiber preserving map, i.e. $\pi \circ \alpha = \pi^*$ where $\pi^*: G^* \times E^n(\rho_1) \rightarrow E^n(\rho_1)$ is the natural projection. Let $\rho \leq \rho_2 = \gamma \rho_1 < \rho_1$ where $0 < \gamma < 1$ is a fixed number. We put $\hat{G}(\rho) = G \times E^n(\rho)$. If f is a holomorphic function on $\hat{G}(\rho)$ we write $f = \sum a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. We define the norm $\|f\|_\rho$ of f by $\|f\|_\rho = \sup_\nu \{ \sup |a_\nu(G)| \}$.

If $f \in I(\hat{G}(\rho))$ we see that $f \circ \alpha$ is a well defined function on $G^* \times E^n(\rho)$ because α is fiber preserving. We define $\|f \circ \alpha\|_\rho$ using G^* instead of G as above. We have the proposition:

Proposition 1. There exists a constant K such that $\|f \circ \alpha\|_\rho \leq K \|f\|_\rho$ where $K = K(\rho_2)$ is independent of $\rho \leq \rho_2$.

Proof. We write $f = \sum_{|\nu|=0}^{\infty} a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. Now we get $f \circ \alpha = \sum (a_\nu \circ \mathfrak{P} \circ \alpha) (t/\rho)^\nu$ because α is fiber preserving. Since $\mathfrak{P}(\hat{G}^*) \subset G$ we get $|a_\nu \circ \mathfrak{P}(\hat{G}^*)| \leq |a_\nu(G)| \leq \|f\|_\rho$. Now $a_\nu \circ \mathfrak{P} \circ \alpha$ admits a Taylor series: $a_\nu \circ \mathfrak{P} \circ \alpha = \sum C_{\nu\lambda} (t/\rho)^\lambda$ with $C_{\nu\lambda} \in I(G^*)$. Since $|\sum C_{\nu\lambda} (t/\rho)^\lambda| \leq \|f\|_\rho$ in $G^* \times E^n(\rho_1)$ and $\rho \leq \rho_2 = \gamma \rho_1$ Cauchy's inequalities give us $|C_{\nu\lambda}(G^*)| \leq \|f\|_\rho \gamma^{|\lambda|}$. Let us put $b_\mu = \sum_{\nu+\lambda=\mu} C_{\nu\lambda}$. We get $|b_\mu(G^*)| \leq \|f\|_\rho \sum \gamma^{|\lambda|} = \|f\|_\rho (1-\gamma)^{-n} = K \|f\|_\rho$. Now we can write $f \circ \alpha = \sum_\nu a_\nu \circ \mathfrak{P} \circ \alpha (t/\rho)^\nu = \sum_{\lambda,\nu} C_{\nu\lambda} (t/\rho)^\lambda (t/\rho)^\nu = \sum_\mu b_\mu (t/\rho)^\mu$. By definition we have $\|f \circ \alpha\|_\rho = \sup_\mu |b_\mu(G^*)| \leq K \|f\|_\rho$.

Let us now consider $\mathbf{h} = (h_{\nu\mu})$ which is a $q \times q$ matrix with $h_{\nu\mu} \in I(\hat{G})$. The $h_{\nu\mu}$ are also assumed to be bounded on \hat{G} .