

Existence of admissible refinements of measure coverings

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$= \{ \hat{U}_i(\rho) \}_1^{i^*}$ is a Stein covering of $X(\rho)$. We say then that $\hat{\mathfrak{U}}(\rho)$ is a measure covering of $X(\rho)$.

Admissible refinements of measure coverings. Let $\hat{\mathfrak{U}}(\rho)$ and $\hat{\mathfrak{U}}^*(\rho)$ be two measure coverings of $X(\rho)$. We say that $\hat{\mathfrak{U}}^*(\rho)$ is an admissible refinement of $\hat{\mathfrak{U}}(\rho)$ if the following conditions hold:

- 1) $U_i^* \subset \subset U_i$ for each i .
- 2) If $U_{i_0 \dots i_\lambda}^* = U_{i_0}^* \cap \dots \cap U_{i_\lambda}^*$ we put $(U_{i_0 \dots i_\lambda}^*)_v = \Phi_v^{-1}(U_{i_0 \dots i_\lambda}^* \times E^n(\rho))$ for each $v \in \{i_0 \dots i_\lambda\}$. It is now required that $(U_{i_0 \dots i_\lambda}^*)_v \subset (U_{i_0 \dots i_\lambda}^*)_\mu$ for all $v, \mu \in \{i_0 \dots i_\lambda\}$.
- 3) $\hat{U}_{i_0 \dots i_\lambda}^* = \hat{U}_{i_0}^* \cap \dots \cap \hat{U}_{i_\lambda}^* \subset (U_{i_0 \dots i_\lambda}^*)_\mu$ for each $\mu \in \{i_0 \dots i_\lambda\}$.

EXISTENCE OF ADMISSIBLE REFINEMENTS OF MEASURE COVERINGS

Existence Theorem. For every fixed integer s we can find, for some $\rho > 0$, a sequence $\mathfrak{U}_s \ll \mathfrak{U}_{s-1} \ll \dots \ll \mathfrak{U}_1 \ll \mathfrak{U}_0$ of finer measure coverings of $X(\rho)$ each of which is an admissible refinement of the following.

Proof. We first construct a measure covering of $X(\rho)$ for some $\rho < \min \rho_i$. Let $\mathfrak{U}_0 = \{ \mathfrak{U}_i \}_1^{i^*}$ be a Stein covering of X_0 such that $U_i \subset \subset W_i$ for $i \in \{1, \dots, i^*\}$. Choose a fixed $\rho_0 < \min \rho_i$. Now the open sets $\Phi_i^{-1}(U_i \times E^n(\rho_0))$ cover X_0 and hence they also cover $X(\rho)$ for some sufficiently small ρ . Hence \mathfrak{U}_0 defines a measure covering of $X(\rho)$. It is also clear that \mathfrak{U}_0 defines a measure covering of $X(\rho')$ for each $\rho' \leq \rho$. Let us now construct \mathfrak{U}_1 . We let $\mathfrak{U}^* = \{ U_i^* \}_1^{i^*}$ be a Stein covering such that $U_i^* \subset \subset U_i$ always holds. Now we can find $\rho_1 \leq \rho$ such that $\{ \hat{U}_i^*(\rho_1) = \Phi_i^{-1}(U_i^* \times E^n(\rho_1)) \}_1^{i^*}$ cover $X(\rho_1)$. Hence $\hat{\mathfrak{U}}^*(\rho_1)$ and $\hat{\mathfrak{U}}(\rho_1)$ are measure coverings of $X(\rho_1)$. But we do not yet know if $\hat{\mathfrak{U}}^*(\rho_1) \ll \hat{\mathfrak{U}}(\rho_1)$. We claim that if $\rho_2 \leq \rho_1$ is sufficiently small then $\hat{\mathfrak{U}}^*(\rho_2) \ll \hat{\mathfrak{U}}(\rho_2)$. For suppose this is false. Say that 2) fails for $\hat{\mathfrak{U}}^*(\rho_2)$ and $\hat{\mathfrak{U}}(\rho_2)$ when $0 < \rho_2 \leq \rho_1$. Hence $\Phi_v^{-1}(U_{i_0 \dots i_\lambda}^* \times E^n(\rho_2)) - \Phi_\mu^{-1}(U_{i_0 \dots i_\lambda} \times E^n(\rho_2))$ are non empty for suitable indices while $\rho_2 \rightarrow 0$. Choose a point x_t from each of these sets. Because $x_t \in X(\rho_1)$ which is relatively compact we may assume that $x_t \rightarrow x_0$. Obviously we get $x_0 \in \overline{U_{i_0 \dots i_\lambda}^*} - U_{i_0 \dots i_\lambda}$, a contradic-

tion because $\overline{U_{\iota_0 \dots \iota_\lambda}^*} \subset \overline{U_{\iota_0}^*} \cap \dots \cap \overline{U_{\iota_\lambda}^*} \subset U_{\iota_0 \dots \iota_\lambda}$. In the same way we can prove that condition 3) is satisfied if ρ_2 is sufficiently small and the theorem is clear.

GENERAL THEORY

Let G be an analytic manifold. We put $\hat{G} = G \times E^n(\rho_1)$ where ρ_1 is an n -tuple of positive numbers. Let $\pi: \hat{G} \rightarrow E^n(\rho_1)$ and $\mathfrak{P}: \hat{G} \rightarrow G$ be the projection maps. $\hat{G}^* \subset \hat{G}$ denotes an open subset and $G^* = \hat{G}^* \cap G \times \{0\}$.

The set G^* can be identified with an open subset of G . We denote by $\alpha: G^* \times E^n(\rho_1) \rightarrow \hat{G}^*$ a biholomorphic fiber preserving map, i.e. $\pi \circ \alpha = \pi^*$ where $\pi^*: G^* \times E^n(\rho_1) \rightarrow E^n(\rho_1)$ is the natural projection. Let $\rho \leq \rho_2 = \gamma \rho_1 < \rho_1$ where $0 < \gamma < 1$ is a fixed number. We put $\hat{G}(\rho) = G \times E^n(\rho)$. If f is a holomorphic function on $\hat{G}(\rho)$ we write $f = \sum a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. We define the norm $\|f\|_\rho$ of f by $\|f\|_\rho = \sup_\nu \{ \sup |a_\nu(G)| \}$.

If $f \in I(\hat{G}(\rho))$ we see that $f \circ \alpha$ is a well defined function on $G^* \times E^n(\rho)$ because α is fiber preserving. We define $\|f \circ \alpha\|_\rho$ using G^* instead of G as above. We have the proposition:

Proposition 1. There exists a constant K such that $\|f \circ \alpha\|_\rho \leq K \|f\|_\rho$ where $K = K(\rho_2)$ is independent of $\rho \leq \rho_2$.

Proof. We write $f = \sum_{|\nu|=0}^{\infty} a_\nu (t/\rho)^\nu$ with $a_\nu \in I(G)$. Now we get $f \circ \alpha = \sum (a_\nu \circ \mathfrak{P} \circ \alpha) (t/\rho)^\nu$ because α is fiber preserving. Since $\mathfrak{P}(\hat{G}^*) \subset G$ we get $|a_\nu \circ \mathfrak{P}(\hat{G}^*)| \leq |a_\nu(G)| \leq \|f\|_\rho$. Now $a_\nu \circ \mathfrak{P} \circ \alpha$ admits a Taylor series: $a_\nu \circ \mathfrak{P} \circ \alpha = \sum C_{\nu\lambda} (t/\rho)^\lambda$ with $C_{\nu\lambda} \in I(G^*)$. Since $|\sum C_{\nu\lambda} (t/\rho)^\lambda| \leq \|f\|_\rho$ in $G^* \times E^n(\rho_1)$ and $\rho \leq \rho_2 = \gamma \rho_1$ Cauchy's inequalities give us $|C_{\nu\lambda}(G^*)| \leq \|f\|_\rho \gamma^{|\lambda|}$. Let us put $b_\mu = \sum_{\nu+\lambda=\mu} C_{\nu\lambda}$. We get $|b_\mu(G^*)| \leq \|f\|_\rho \sum \gamma^{|\lambda|} = \|f\|_\rho (1-\gamma)^{-n} = K \|f\|_\rho$. Now we can write $f \circ \alpha = \sum_\nu a_\nu \circ \mathfrak{P} \circ \alpha (t/\rho)^\nu = \sum_{\lambda,\nu} C_{\nu\lambda} (t/\rho)^\lambda (t/\rho)^\nu = \sum_\mu b_\mu (t/\rho)^\mu$. By definition we have $\|f \circ \alpha\|_\rho = \sup_\mu |b_\mu(G^*)| \leq K \|f\|_\rho$.

Let us now consider $\mathbf{h} = (h_{\nu\mu})$ which is a $q \times q$ matrix with $h_{\nu\mu} \in I(\hat{G})$. The $h_{\nu\mu}$ are also assumed to be bounded on \hat{G} .