## Approximation

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## Approximation

We use positive $n$-tuples $\rho, \ldots$ with $\rho \leqslant \rho_{2}<\rho_{3}<\rho_{4}<\rho_{1}$ and $\rho=$ $=\gamma^{\prime \prime} \rho_{1}, \rho_{2}=\gamma \rho_{1}, \rho_{3}=\gamma^{\prime} \rho_{1}, \rho_{4}=\gamma^{\prime \prime} \rho_{1}$. The $n$-tuple $\rho_{1}$ is defined as in the smoothing theorem.

Definition: $H_{*}^{l}=\left\{\xi \in H^{l}\left(X_{0}, F \mid X_{0}\right)\right.$ such that there exists $U=$ $=U(0)$ in $E^{n}$ with $\hat{\xi} \in H^{l}\left(\psi^{-1}(U), \bar{F}\right)$ and $\left.\hat{\xi} \mid X_{0}=\xi\right\}$. Serre's theorem gives $\operatorname{dim}_{\mathrm{C}} H_{*}^{l} \leqslant \operatorname{dim}_{\mathrm{C}} H^{l}\left(X_{0}, F \mid X_{0}\right)<\infty$. In the following discussion we are given $\hat{\mathfrak{b}}_{1}, \ldots, \hat{\mathfrak{b}}_{r}$ in $Z^{l}\left(\hat{\mathfrak{U}^{\prime}}\left(\rho_{4}\right), \mathbf{F}\right)$ such that $\hat{\mathfrak{b}}_{1}\left|X_{0}, \ldots \hat{\mathfrak{b}}_{r}\right| X_{0}$ constitute a base of the complex vector space $H_{*}^{l}$. For this to be possible, $\rho_{4}$ has to be chosen small enough. Here $\mathfrak{X} \mathfrak{X}^{\prime}$ is a Stein covering of $X\left(\rho_{1}\right)$ and defined as in the smoothing theorem. We also assume that we are given a sequence of measure coverings as there. Further we construct the sequence so that there are still sufficiently many measure coverings in between $\mathfrak{B}$ and $\mathfrak{U}$. These are denoted by $\mathfrak{U}_{v}^{*}$. We have $\mathfrak{U} \gg \mathfrak{U}_{1}^{*} \gg \mathfrak{U}_{2}^{*} \gg \ldots \gg \mathfrak{B}$. The $n$-tupel $\rho_{3}$ is also fixed from now on and $K$ always denotes (possibly different) constants.

Approximation Lemma: Let $\varepsilon>0$. Then we can find $\rho_{2}$ such that: If $\rho \leqslant \rho_{2}$ and $\hat{\xi} \in Z^{l}(\hat{\mathfrak{U}}(\rho), \mathbf{F})$ with $\|\hat{\xi}\|_{\rho}<\infty$ (the norm is taken with respect to $\left.\hat{\mathfrak{U}}_{1}^{*}(\rho)\right)$, then there exist $a_{1}, \ldots a_{r} \in I\left(E^{n}(\rho)\right)$ and $\hat{\eta} \in C^{l-1}(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ such that $\tilde{\xi}=\hat{\xi}-\sum_{1}^{r} a_{i} \hat{\mathfrak{b}}_{i}-\hat{\delta \eta}$ on $\hat{\mathfrak{B}}(\rho)$. Here $\tilde{\xi} \in Z^{l}(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ and $\|\tilde{\xi}\|_{\rho} \leqslant$ $\leqslant \varepsilon\|\hat{\xi}\|_{\rho}$ and $\left\|a_{v}\right\|_{\rho},\|\hat{\eta}\|_{\rho} \leqslant K\|\hat{\xi}\|_{\rho} . K$ is a fixed constant.

Proof. We shall first prove some results which are needed later on. Let $S \in \Gamma\left(\hat{U}_{\iota_{0} \ldots l_{l}}(\rho), \mathbf{F}\right)$. Choose $\iota \in\left\{\iota_{0}, \ldots, \iota_{l}\right\}$. Now $\left(U_{\iota_{0} \ldots l_{l}}^{(1)^{*}}\right) \subset \hat{U}_{\iota_{0} \ldots l_{l}}$ because $\mathfrak{l}_{1}^{*} \ll \mathfrak{U}$. The operations are always defined with respect to $\rho_{1}$. We can now restrict $S$ to $\left(U_{\iota_{0} \ldots \iota_{l}}^{(1)_{l}^{*}}\right)_{l}(\rho)$. In the chart $\mathscr{W}$, we can write $S=$ $=\sum a_{v}(t / \rho)^{\nu}$. Here $a_{v} \in q I\left(U_{\iota_{0} \ldots{ }_{l}}^{(1)_{*}, \ldots}\right)$. Now the $a_{v}$ are extended constantly and we get elements $\hat{a}_{v} \in \Gamma\left(\left(U_{\iota_{0} \ldots l_{l}}^{(1)^{*}}\right)_{v}, \mathbf{F}\right)$. Let us put $S_{v}=\hat{a}_{v} \mid \hat{U}_{\iota_{0}}^{(2)^{*}} \ldots \iota_{l}$. We claim that $\left\|S_{v}\right\|_{\rho_{1}} \leqslant K\|S\|_{\rho}$. For obviously $\|S\|_{\rho} \geqslant\left|a_{v}\left(U_{\iota_{0} \ldots \iota l}^{(1)^{*}}\right)\right|$ and
we can use the Theorem I to prove that $\left\|S_{v}\right\|_{\rho_{1}} \leqslant K\left\|\hat{a_{v}} \mid\left(U_{L_{0} \ldots l_{l}}^{(1)^{*}}\right)_{\iota}\left(\rho_{1}\right)\right\|_{1}=$ $=K\left|a_{v}\left(U_{\iota_{0} \ldots \mu_{l}}^{(1)^{*}}\right)\right| \leqslant K\|S\|_{\rho}$. Q.E.D.

Let $S_{v}^{\prime}$ be defined using some other $\iota^{\prime} \in\left\{\iota_{0}, \ldots \iota_{l}\right\}$. Then $S_{v}-S_{v}^{\prime} \in$ $\in \Gamma\left(\hat{U}_{\iota_{0} \cdots l^{\prime}}^{(2)^{*}}, \mathbf{F}\right)$. We claim that $\left\|S_{v}-S_{v}^{\prime}\right\|_{\rho_{4}} \leqslant K \gamma^{\prime \prime \prime}\|S\|_{\rho}$.

Proof. Define $\alpha_{s}=\sum_{|\lambda|=s}^{\infty} a_{\lambda}(t / \rho)^{\lambda}$ and $\beta_{s}=\sum_{|\lambda|=0}^{s-1} a_{\lambda}(t / \rho)^{\lambda} \quad$ over $\left(U_{\iota 0}^{\left.(1)^{*} \ldots, l_{l}\right)}\right)_{\iota}(\rho)$. We do the same for $\iota^{\prime}$ respectively and obtain $\alpha_{s}^{\prime}$ and $\beta_{s}^{\prime}$ over $\left(U_{L_{0} \ldots l_{l}}^{(1)^{*}}\right)_{L^{\prime}}(\rho)$. For the restrictions to $\hat{U}_{\iota_{0} \ldots l_{l}}^{(2)^{*}}$ we see that $\alpha_{s}-\alpha_{s}^{\prime}=$ $-\left(\beta_{s}-\beta_{s}^{\prime}\right)$. Hence we get $\left\|\alpha_{s}-\alpha_{s}^{\prime}\right\|_{\rho_{4}} \leqslant K\left(\gamma^{\prime \prime \prime}\right)^{s}\left\|\alpha_{s}-\alpha_{s}^{\prime}\right\|_{\rho_{1}}=K\left(\gamma^{\prime \prime \prime}\right)^{s} \| \beta_{s}-$ $-\beta_{s}\left\|_{\rho_{1}} \leqslant K\left(\gamma^{\prime \prime \prime}\right)^{s}\right\| \beta_{s}\left\|_{\rho_{1}}+K\left(\gamma^{\prime \prime \prime}\right)^{s}\right\| \beta_{s}^{\prime} \|_{\rho_{1}} \leqslant K\left(\gamma^{\prime \prime \prime}\right)^{s}\left[\left\|\beta_{s}\right\|_{\rho_{1}}{ }^{*}+\left\|\beta_{s}^{\prime}\right\|_{\rho_{1}}{ }^{*}\right] \leqslant$ $\leqslant K\left(\gamma^{\prime \prime \prime}\right)^{s}\left(\gamma^{\prime \prime}\right)^{1-s}\|S\|_{\rho}$. Here the norms are defined with respect to $U_{L_{0}}^{(3) * \psi_{l}^{*}}$ except $\left\|\|^{*}\right.$ and $\| S \|_{\rho}$ which are defined with respect to $U_{\iota_{0} \cdots l_{l}}^{(1)^{*}}$. Now we look at the difference $\left(S_{v}-S_{v}^{\prime}\right) t^{v} / \rho^{v}$ on $\left(U_{L_{0}}^{(3)^{*} \ldots l_{l}}\right)_{\mu}$ with $|v|=s, \mu \in\left\{\iota_{0}, \ldots L_{l}\right\}$, and the power series development with respect to $W_{\mu}$. There is one term of order $s$ which is equal to the corresponding term of $\alpha_{s}-\alpha_{s}^{\prime}$. Therefore its norm is $\leqslant K\left(\gamma^{\prime \prime \prime}\right)^{s} .\left(\gamma^{\prime \prime}\right)^{1-s}\|S\|_{\rho}$. Moreover we have $\left\|S_{v}(t / \rho)^{v}-S_{v}^{\prime}(t / \rho)^{v}\right\|_{\rho_{1}} \leqslant$ $\leqslant\left(\gamma^{\prime \prime}\right)^{-s} \cdot K\|S\|_{\rho}$ where the first norm is defined with respect to $U_{{ }_{6}}^{(3)^{*} \ldots u}$. For the sum $\sum$ of terms of higher order than $s$ in the power series of $\left(S_{v}-\right.$ $\left.-S_{v}^{\prime}\right) t^{v} / \rho^{v}$ we therefore get: $\left\|\sum\right\|_{\rho_{4}} \leqslant\left(\gamma^{\prime \prime \prime}\right)^{s+1}\left(\gamma^{\prime \prime}\right)^{-s} . K\|S\|_{\rho}$. Hence we get $\left\|\left(S_{v}-S_{v}^{\prime}\right)\right\|_{\rho_{4}} \leqslant \gamma^{\prime \prime \prime} \cdot K\|S\|_{\rho}$. This proves our statement. We see that $K$ is independent of $\rho_{4}$ and $S$. The number $\gamma^{\prime \prime \prime}$ depends on $\rho_{4}$ only, so $\gamma^{\prime \prime \prime} \cdot K$ gets very small if we make $\rho_{4}$ very small.

Let $\hat{\xi} \in Z^{l}(\hat{\mathfrak{U}}(\rho), \mathbf{F})$ with $\hat{\xi}=\left\{\hat{\xi}_{l_{0} \ldots l_{l}}\right\}$. Choose $i=l\left(\iota_{0}, \ldots, l_{l}\right)$ as a function of the unordered $(l+1)$-tuple. We now fix $\iota_{0}, \ldots, l_{l}$ and write $S=\hat{\xi}_{l_{0} \ldots l l}$. We apply to $S$ the method described above and obtain $\hat{\xi}_{l_{0} \ldots l l}^{(v)}=$ $=S_{v}$. We do this now for every $\iota_{0}, \ldots, l_{l}$ and consider $\hat{\xi}_{(v)}=\left\{\hat{\xi}_{\nu_{0} \ldots l l}^{(v)}\right\}$ as an element of $C^{l}\left(\hat{\mathfrak{U}}_{2}^{*}\left(\rho_{4}\right), \mathbf{F}\right)$. Of course $\hat{\xi}_{(v)}$ depends on the choice of $t=\imath\left(\iota_{0} \ldots l_{l}\right)$ here. Now we see that $\left\|\hat{\xi}_{(v)}\right\|_{\rho_{4}} \leqslant\left\|\hat{\xi}_{(v)}\right\|_{\rho_{1}} \leqslant K\|\hat{\xi}\|_{\rho}$. We also wish to estimate $\hat{\xi}_{(v)}$. Because $\hat{\xi} \in Z^{l}(\hat{\mathfrak{U}}(\rho), \mathbf{F})$ we can use the preliminary result on $\iota$ and $\iota^{\prime}$ to obtain $\left\|\delta \hat{\xi}_{(\nu)}\right\|_{\rho_{4}} \leqslant K \gamma^{\prime \prime \prime}\|\hat{\xi}\|_{\rho}$.

We shall also need another result:

Induction Lemma: There exists $\hat{\eta}_{v} \in C^{l}\left(\hat{\mathfrak{U}}_{4}^{*}\left(\rho_{3}\right), \mathbf{F}\right)$ such that $\hat{\delta \eta_{v}}=$ $=\delta \hat{\xi}_{(v)}$ on $\hat{\mathfrak{U}}_{4}^{*}\left(\rho_{3}\right)$ and $\left\|\hat{\eta}_{v}\right\|_{\rho_{3}} \leqslant K\left\|\delta \hat{\xi}_{(v)}\right\|_{\rho_{4}}$.

Proof. The proof uses the assumption that $\psi_{(l+1)}(\mathbf{F})$ is coherent. Because the coherence of direct images is proved by downward induction on $l$, this assumption can be made. Moreover it is assumed that the main theorem is proved for dimension $l+1$ already. Let us now put $\alpha=\delta \hat{\xi}_{(\nu)} \in$ $\in B^{l+1}\left(\hat{\mathfrak{U}}_{2}^{*}\left(\rho_{4}\right), \mathbf{F}\right)$ and $\hat{\eta_{v}}=\beta \in C^{l}\left(\hat{\mathfrak{U}}_{4}^{*}\left(\rho_{3}\right), \mathbf{F}\right)$. We have to prove the existence of $\beta$. We may assume that $\rho_{4}$ is so small that the main theorem is valid for $\rho \leqslant \rho_{4}$ in the case of dimension $l+1$. So there are cocycles $\omega_{1}, \ldots, \omega_{r} \in Z^{l+1}\left(\hat{\mathfrak{U}}\left(\rho_{4}\right), \mathbf{F}\right)$ such that $\alpha=\sum C_{\lambda} \omega_{\lambda}+\delta \eta$, where $C_{\lambda} \in$ $\in I\left(E^{n}\left(\rho_{4}\right)\right)$ and $\eta \in C^{l}\left(\hat{\mathfrak{U}}_{4}^{*}\left(\rho_{4}\right), \mathbf{F}\right)$. We have to assume that between $\hat{\mathfrak{U}}_{4}^{*}$ and $\mathfrak{U}_{2}^{*}$ there are very many measure coverings. The cross-sections $\psi_{(l+1)}\left(\omega_{\lambda}\right)$ give a homomorphism $r \mathcal{O} \rightarrow \psi_{(l+1)}(\mathbf{F})$ over $E^{n}\left(\rho_{4}\right)$. Because $\psi_{(l+1)}(\mathbf{F})$ is coherent the kernel $\mathscr{N}$ is coherent again. Over $E^{n}\left(\rho^{\prime}\right)$ with $\rho_{3}<\rho^{\prime}<\rho_{4}$ we find an epimorphism $p \mathcal{O} \rightarrow \mathscr{N}$. Denote by $n_{1}, \ldots, n_{p}$ the images of the unit cross-sections in $p \mathcal{O}$. Write $n_{\lambda}=\left(e_{\lambda 1}, \ldots, e_{\lambda r}\right)$ as an $r$-tupel of holomorphic functions. The image of $n_{\lambda}$ in $\Gamma\left(E^{n}\left(\rho^{\prime}\right), \psi_{(l+1)}(\mathbf{F})\right)$ is $\psi_{(l+1)}\left(\sum_{\mu=1}^{r} e_{\lambda \mu} \omega_{\mu}\right)$ and zero. We may choose $\rho_{2}$ and then $\rho_{3}$ and $\rho^{\prime}$ very small. Then it follows that $\hat{n}_{\lambda}=\sum e_{\lambda \mu} \omega_{\mu}$ is a coboundary. If $\rho_{3}<\rho^{\prime \prime}<\rho^{\prime}$ there are cochains $\eta_{\lambda} \in C^{l}\left(\hat{\mathfrak{U}}_{4}^{*}\left(\rho^{\prime \prime}\right), \mathbf{F}\right)$ such that $\delta \eta_{\lambda}=\hat{n}_{\lambda}$. Now $\left(C_{1}, \ldots, C_{r}\right) \in$ $\in \Gamma\left(E^{n}\left(\rho_{4}\right), \mathscr{N}\right)$. By the methods of sheaf theory we can lift this crosssection to $p \mathcal{O}$. Using a " Banach open mapping theorem " we see that the map $\Gamma\left(E^{n}\left(\rho^{\prime}\right), p \mathscr{\theta}\right) \rightarrow \Gamma\left(E^{n}\left(\rho^{\prime}\right), \mathscr{N}\right)$ is open. This means here that we can find holomorphic functions $a_{\lambda}$ over $E^{n}\left(\rho_{3}\right)$ such that $C_{\mu}=\sum a_{\lambda} e_{\lambda \mu}$ and $\left\|a_{\lambda}\right\|_{\rho_{3}} \leqslant K \max \left\|C_{\mu}\right\|_{\rho^{\prime}} \leqslant K \max \left\|C_{\mu}\right\|_{\rho_{4}}$. We get $\sum C_{\mu} \omega_{\mu}=\sum a_{\lambda} e_{\lambda \mu} \omega_{\mu}$ $=\sum a_{\lambda} \hat{n_{\lambda}}=\delta\left(\sum a_{\lambda} \eta_{\lambda}\right)$. This leads to $\alpha \mid C^{l+1}\left(\hat{\mathfrak{l}}_{4}^{*}\left(\rho_{3}\right)\right)=\delta\left(\eta+\sum a_{\lambda} \eta_{\lambda}\right)$. The estimates required obviously hold. Q.E.D.

Let us now put $\hat{\xi}_{(v)}^{*}=\hat{\xi}_{(v)}-\hat{\eta}_{v} \in Z^{l}\left(\hat{\mathfrak{U}}_{4}\left(\rho_{3}\right), \mathbf{F}\right)$. We can write $\hat{\xi}_{(v)}^{*} \mid X_{0}=$ $=\sum a_{\nu \lambda} \hat{\mathfrak{b}}_{\lambda} \mid X_{0}+\delta \gamma_{v}$ over $\mathfrak{l}_{6}^{*}$. Here $a_{v \lambda}$ are complex numbers and $\gamma_{v} \in$ $\in C^{l-1}\left(\mathfrak{U}_{6}^{*}, F \mid X_{0}\right)$. Cartan's theorem and the result after that give the estimates $\overline{\left|a_{v \lambda}\right|} \leqslant K\left\|\hat{\xi}_{(v)}^{*}\right\|_{\rho_{3}} \leqslant K\|\hat{\xi}\|_{\rho}$ and $\left\|\hat{\gamma}_{v}\right\|_{\rho_{3}} \leqslant K\left\|\hat{\xi}_{(v)}^{*}\right\|_{\rho_{3}} \leqslant$ $\leqslant K\|\hat{\xi}\|_{\rho}$. Here $\hat{\gamma_{v}} \in C^{l-1}\left(\hat{\mathfrak{U}}_{7}^{*}\left(\rho_{3}\right), \mathbf{F}\right)$ has been obtained by a constant
extension of $\gamma_{v}$. Let us now put $\hat{\xi}_{(v)}^{(1)}=\hat{\xi}_{(v)}^{*}-\sum a_{v \lambda} \hat{b}_{\lambda}-\delta \hat{\gamma}_{v}$. Here $\hat{\xi}_{(v)}^{(1)} \in$ $\in C^{l}\left(\hat{\mathfrak{u}}_{7}^{*}\left(\rho_{3}\right), \mathbf{F}\right)$. Using the previous estimates and the fact that the $\hat{\mathfrak{b}}_{\lambda}$ are finite we find that $\left\|\hat{\xi}_{(v)}^{(1)}\right\|_{\rho_{3}} \leqslant K\left\|\hat{\xi}_{(v)}\right\|_{\rho_{4}} \leqslant K\|\hat{\xi}\|_{\rho}$.

Now we also have $\hat{\xi}_{(\nu)}^{(1)} \mid X_{0}=0$. It follows that

$$
\left\|\hat{\xi}_{(v)}^{(1)}\right\|_{\rho} \leqslant \gamma / \gamma^{\prime}\left\|\hat{\xi}_{(v)}^{(1)}\right\|_{\rho_{3}} \leqslant \gamma / \gamma^{\prime} \cdot K\|\hat{\xi}\|_{\rho}
$$

Finally we put in $\hat{\mathfrak{U}}_{9}^{*}(\rho)$ :

$$
\begin{gathered}
\hat{\xi}^{(1)}=\Sigma \hat{\xi}_{(v)}^{(1)}(t / \rho)^{v}= \\
=\Sigma \hat{\xi}_{(v)}(t / \rho)^{v}-\Sigma \hat{\eta}_{v}(t / \rho)^{v}-\Sigma a_{v \lambda}(t / \rho)^{v} \hat{\mathfrak{b}}_{\lambda}-\delta\left(\Sigma \hat{\gamma}_{\nu}(t / \rho)^{v}\right) \\
=\hat{\xi}-\hat{\eta}-\Sigma a_{\lambda} \hat{\mathfrak{b}}_{\lambda}-\hat{\delta \gamma}
\end{gathered}
$$

Using the fact that the sum of the absolute values of the coefficients in the power series expansion of $\hat{\xi}_{(v)}^{(1)}$ by $(t / \rho)$ is smaller than $\gamma / \gamma^{\prime} \cdot K\|\hat{\xi}\|_{\rho}$ and that with respect to $\hat{\eta}_{\nu}$ is smaller than $\gamma^{\prime \prime \prime} \cdot K\|\hat{\xi}\|_{\rho}$ we find: $\left\|\hat{\xi}^{(1)}\right\|_{\rho} \leqslant$ $\leqslant \gamma / \gamma^{\prime} \cdot K\|\hat{\xi}\|_{\rho}$ and $\|\hat{\eta}\|_{\rho} \leqslant \gamma^{\prime \prime \prime} \cdot K\|\hat{\xi}\|_{\rho}$ and $\left\|a_{\lambda}\right\|_{\rho} \leqslant K\|\hat{\xi}\|_{\rho}$. We take the restriction to $\hat{\mathfrak{V}}(\rho)$ and now $\tilde{\xi}=\hat{\xi}^{(1)}-\hat{\eta} \in Z^{l}(\hat{\mathfrak{V}}(\rho), \mathbf{F})$ is the desired element. Of course we have to choose $\rho_{4}$ and then $\rho_{2}$ small enough, for example let $\gamma^{\prime \prime \prime}<\varepsilon / 2 K$ and $\gamma \leqslant \varepsilon \gamma^{\prime} / 2 K$.

## Main Theorem

There exists $\rho_{2}$ and a constant $K$ such that if $\rho \leqslant \rho_{2}$ and $\hat{\xi} \in Z^{l}(\hat{\mathfrak{U}}(\rho), \mathbf{F})$ with $\|\hat{\xi}\|_{\rho}<\infty$ then we can find $a_{1}, \ldots, a_{r} \in I\left(E^{n}(\rho)\right)$ and $\hat{\eta} \in$ $C^{l-1}(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ such that $\hat{\xi}=\sum a_{\lambda} \hat{\mathfrak{b}}_{\lambda}+\hat{\delta \eta}$ on $\hat{\mathfrak{B}}(\rho)$ with $\|\hat{\eta}\|_{\rho}$ and $\left\|a_{v}\right\|_{\rho} \leqslant$ $\leqslant K\|\hat{\xi}\|_{\rho}$.

Proof. We have one constant $K$ from the smoothing theorem. Now we find $\rho_{2}$ with an $\varepsilon$ in the Approximation Lemma such that $\varepsilon \cdot K<1 / 2$. We shall use this $\rho_{2}$ and prove the theorem here. We are given $\hat{\xi}_{0}=\hat{\xi} \in$ $Z^{l}(\hat{\mathfrak{l}}(\rho), \mathbf{F})$ with $\|\hat{\xi}\|_{\rho}<\infty$. The Approximation Lemma gives $\tilde{\xi}_{1}=$

