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Autor: Grauert, H.
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extension of γ_v . Let us now put $\hat{\xi}_{(v)}^{(1)} = \hat{\xi}_{(v)}^* - \sum a_{v\lambda} \hat{b}_\lambda - \delta \hat{\gamma}_v$. Here $\hat{\xi}_{(v)}^{(1)} \in C^l(\hat{\mathfrak{U}}_7^*(\rho_3), \mathbb{F})$. Using the previous estimates and the fact that the \hat{b}_λ are finite we find that $\|\hat{\xi}_{(v)}^{(1)}\|_{\rho_3} \leq K \|\hat{\xi}_{(v)}\|_{\rho_4} \leq K \|\hat{\xi}\|_{\rho}$.

Now we also have $\hat{\xi}_{(v)}^{(1)}|_{X_0} = 0$. It follows that

$$\|\hat{\xi}_{(v)}^{(1)}\|_{\rho} \leq \gamma/\gamma' \|\hat{\xi}_{(v)}^{(1)}\|_{\rho_3} \leq \gamma/\gamma' \cdot K \|\hat{\xi}\|_{\rho}.$$

Finally we put in $\hat{\mathfrak{U}}_9^*(\rho)$:

$$\begin{aligned} \hat{\xi}^{(1)} &= \sum \hat{\xi}_{(v)}^{(1)} (t/\rho)^v = \\ &= \sum \hat{\xi}_{(v)} (t/\rho)^v - \sum \hat{\eta}_v (t/\rho)^v - \sum a_{v\lambda} (t/\rho)^v \hat{b}_\lambda - \delta (\sum \hat{\gamma}_v (t/\rho)^v) \\ &= \hat{\xi} - \hat{\eta} - \sum a_\lambda \hat{b}_\lambda - \delta \hat{\gamma}. \end{aligned}$$

Using the fact that the sum of the absolute values of the coefficients in the power series expansion of $\hat{\xi}_{(v)}^{(1)}$ by (t/ρ) is smaller than $\gamma/\gamma' \cdot K \|\hat{\xi}\|_{\rho}$ and that with respect to $\hat{\eta}_v$ is smaller than $\gamma''' \cdot K \|\hat{\xi}\|_{\rho}$ we find: $\|\hat{\xi}^{(1)}\|_{\rho} \leq \gamma/\gamma' \cdot K \|\hat{\xi}\|_{\rho}$ and $\|\hat{\eta}\|_{\rho} \leq \gamma''' \cdot K \|\hat{\xi}\|_{\rho}$ and $\|a_\lambda\|_{\rho} \leq K \|\hat{\xi}\|_{\rho}$. We take the restriction to $\hat{\mathfrak{B}}(\rho)$ and now $\tilde{\xi} = \hat{\xi}^{(1)} - \hat{\eta} \in Z^l(\hat{\mathfrak{B}}(\rho), \mathbb{F})$ is the desired element. Of course we have to choose ρ_4 and then ρ_2 small enough, for example let $\gamma''' < \varepsilon/2 K$ and $\gamma \leq \varepsilon\gamma'/2 K$.

MAIN THEOREM

There exists ρ_2 and a constant K such that if $\rho \leq \rho_2$ and $\hat{\xi} \in Z^l(\hat{\mathfrak{U}}(\rho), \mathbb{F})$ with $\|\hat{\xi}\|_{\rho} < \infty$ then we can find $a_1, \dots, a_r \in I(E^n(\rho))$ and $\hat{\eta} \in C^{l-1}(\hat{\mathfrak{B}}(\rho), \mathbb{F})$ such that $\hat{\xi} = \sum a_\lambda \hat{b}_\lambda + \delta \hat{\eta}$ on $\hat{\mathfrak{B}}(\rho)$ with $\|\hat{\eta}\|_{\rho}$ and $\|a_v\|_{\rho} \leq K \|\hat{\xi}\|_{\rho}$.

Proof. We have one constant K from the smoothing theorem. Now we find ρ_2 with an ε in the Approximation Lemma such that $\varepsilon \cdot K < 1/2$. We shall use this ρ_2 and prove the theorem here. We are given $\hat{\xi}_0 = \hat{\xi} \in Z^l(\hat{\mathfrak{U}}(\rho), \mathbb{F})$ with $\|\hat{\xi}\|_{\rho} < \infty$. The Approximation Lemma gives $\tilde{\xi}_1 =$

$= \hat{\xi} - \sum a_{1\lambda} \hat{\mathbf{b}}_\lambda - \hat{\delta}\gamma_1$ on $\hat{\mathfrak{B}}(\rho)$. Here $\hat{\gamma}_1 \in C^{l-1}(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ and $\|\hat{\xi}_1\|_\rho \leq \varepsilon \|\hat{\xi}\|_\rho$. Now $\hat{\xi}_1 \in Z^l(\hat{\mathfrak{B}}(\rho), \mathbf{F})$. The Smoothing Theorem gives $\hat{\xi}_1 \in Z^l(\hat{\mathfrak{U}}(\rho), \mathbf{F})$ and $\hat{\eta}_1 \in C^{l-1}(\hat{\mathfrak{B}}(\rho), \mathbf{F})$ such that $\hat{\xi}_1 = \tilde{\xi}_1 + \hat{\delta}\eta_1$ on $\hat{\mathfrak{B}}(\rho)$. Here $\|\eta_1\|_\rho$ and $\|\hat{\xi}_1\|_\rho \leq K \|\tilde{\xi}_1\|_\rho < 1/2 \|\hat{\xi}\|_\rho$. Now we use $\hat{\xi}_1$ instead of $\hat{\xi}_0$ as above and get: $\hat{\xi}_2 = \hat{\xi}_1 + \hat{\delta}\eta_2 - \sum a_{2\lambda} \hat{\mathbf{b}}_\lambda - \hat{\delta}\gamma_2$. Here $\|\hat{\xi}_2\|_\rho$ and $\|\hat{\eta}_2\|_\rho < 1/2 \|\hat{\xi}_1\|_\rho < (1/2)^2 \|\hat{\xi}\|_\rho$ and $\|a_{2\lambda}\|_\rho$ and $\|\gamma_2\|_\rho \leq \frac{K}{2} \|\hat{\xi}\|_\rho$. Inductively we get: $\hat{\xi}_n = \hat{\xi}_{n-1} - \sum a_{n\lambda} \hat{\mathbf{b}}_\lambda - \hat{\delta}\gamma_n + \hat{\delta}\eta_n$. Here $\|\hat{\xi}_n\|_\rho < 2^{-n} \|\hat{\xi}\|_\rho$, $\|\hat{\eta}_n\|_\rho \leq 2^{-n} \|\hat{\xi}\|_\rho$ and $\|a_{n\lambda}\|_\rho$ and $\|\gamma_n\|_\rho \leq 2^{-n+1} \cdot K \|\hat{\xi}\|_\rho$ for $n = 1, 2, 3, \dots$. A summation is now possible. We get $0 = \hat{\xi} - \sum_{n,\lambda} a_{n\lambda} \hat{\mathbf{b}}_\lambda - \sum \hat{\delta}\gamma_n + \sum \hat{\delta}\eta_n$. We put $a_\lambda = \sum_n a_{n\lambda}$, $\hat{\eta} = \sum (-\hat{\gamma}_n + \hat{\eta}_n)$ and the theorem follows.

For the proof of the coherence the Main Theorem is needed in a weaker and simpler form.

Main Theorem ()*: There exists a positive n -tuple $\rho_2 \leq \rho_0$ and cross-sections $S_1, \dots, S_r \in \Gamma(E^n(\rho_2), \psi_{(l)}(\mathbf{F}))$ such that any $S = \psi_{(l)}(\hat{\xi}') \in \Gamma(E^n(\rho'), \psi_{(l)}(\mathbf{F}))$ with $\hat{\xi}' \in H^l(X(\rho'), \mathbf{F})$ can be written over $E^n(\rho)$ in the form $S = \sum_1^n a_\lambda S_\lambda$ with $a_1, \dots, a_r \in I(E^n(\rho))$. Here $\rho \leq \rho_2$ and $\rho < \rho' \leq \rho_0$.

Proof. Define $S_\lambda = \psi_{(l)}(\hat{\mathbf{b}}_\lambda | X(\rho_2))$. The cross-section S can be written in the form $S = \psi_{(l)}(\hat{\xi}')$ with $\hat{\xi}' \in Z^l(\hat{\mathfrak{U}}'(\rho'), \mathbf{F})$. We put $\hat{\xi} = \hat{\xi}' | \mathfrak{U}(\rho)$. Then $\|\hat{\xi}\|_\rho < \infty$ and we have the representation $\hat{\xi} = \sum a_\lambda \hat{\mathbf{b}}_\lambda + \hat{\delta}\eta$. For the cohomology classes we get $\hat{\xi} = \sum a_\lambda \hat{\mathbf{b}}_\lambda$ and for the images $S | E^n(\rho)$, this gives $S | E^n(\rho) = \psi_{(l)}(\hat{\xi}) = \sum a_\lambda S_\lambda$.

The immediate consequence of this form of the Main Theorem is that the stalk of $\psi_{(l)}(\mathbf{F})$ at the origin (and hence at every point of course) is finitely generated. However this is not yet the full coherence of $\psi_{(l)}(\mathbf{F})$. Nevertheless, the Main Theorem above contains all that is essential, and the rest of the proof is not difficult. We refer to [1, pp. 54-58], or to Knorr [2] for details.