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Artikel: AN ELEMENTARY PROOF THAT ELLIPTIC CURVES ARE ABELIAN VARIETIES

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Bibliographie

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Corollary 13 Given the situation in Lemma 12, assume also that for some $x_0 \in X(k)$, $f(\{x_0\} \times Y)$ is the point z_0. Then $f(X \times Y) = \{z_0\}$.

Proof:

By the rigidity lemma, there exists $g : Y \to Z$ such that $f = g \circ p_2$. $f(x, y) = (g \circ p_2)(x, y) = g(y) = (g \circ p_2)(x_0, y) = f(x_0, y) = z_0$.

Corollary 14 If X and Y are abelian varieties and $f : X \to Y$ is any morphism, then there exists a homomorphism $h : X \to Y$ and a k-point $a \in Y(k)$ such that $f = T_a \circ h$ where T_a denotes translation by a.

Corollary 15 Let X and Y be abelian varieties. Then X and Y are isomorphic as abelian varieties $\Leftrightarrow$ X and Y are isomorphic as schemes.

Proof:

($\Rightarrow$.) obvious

($\Leftarrow$.) Let $f : X \to Y$ be an isomorphism of schemes. f can be written as $f = Y_a \circ h$ with $a \in Y(k)$ and h a homomorphism. T_a is an isomorphism of schemes with T_{Ua} as its inverse. Therefore $h = T_{Ua} \circ f$ is an isomorphism of schemes and hence of abelian varieties.

Corollary 16 Let X be a variety and suppose that (X, m) and (X, m') are two abelian variety structures on X with identity elements e and e' respectively. Then m and m' differ only by translation.

Proof:

Let $+$, $-$, and translation all denote operations with respect to m. Consider the morphism $(m-m') : X \times X \to X$. We have $(m-m')(X \times \{e'\}) = e' = (m-m')(\{e'\} \times X)$. By Corollary 13, $(m-m')(X \times Y) = e'$, i.e. $m = m' + \text{e}'$.

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