

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 19 (1973)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: AN ELEMENTARY PROOF THAT ELLIPTIC CURVES ARE ABELIAN VARIETIES
Autor: Olson, Loren D.
Bibliographie
DOI: <https://doi.org/10.5169/seals-46291>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 18.04.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

Corollary 13 Given the situation in Lemma 12, assume also that for some $x_0 \in X(k)$, $f(\{x_0\} \times Y)$ is the point z_0 . Then $f(X \times Y) = \{z_0\}$.

Proof:

By the rigidity lemma, there exists $g : Y \rightarrow Z$ such that $f = g \circ p_2$.
 $f(x, y) = (g \circ p_2)(x, y) = g(y) = (g \circ p_2)(x_0, y) = f(x_0, y) = z_0$.

Corollary 14 If X and Y are abelian varieties and $f : X \rightarrow Y$ is any morphism, then there exists a homomorphism $h : X \rightarrow Y$ and a k -point $a \in Y(k)$ such that $f = T_a \circ h$ where T_a denotes translation by a .

Corollary 15 Let X and Y be abelian varieties. Then X and Y are isomorphic as abelian varieties $\Leftrightarrow X$ and Y are isomorphic as schemes.

Proof:

($\Rightarrow$.) obvious

($\Leftarrow$.) Let $f : X \rightarrow Y$ be an isomorphism of schemes. f can be written as $f = Y_a \circ h$ with $a \in Y(k)$ and h a homomorphism. T_a is an isomorphism of schemes with T_{-a} as its inverse. Therefore $h = T_{-a} \circ f$ is an isomorphism of schemes and hence of abelian varieties.

Corollary 16 Let X be a variety and suppose that (X, m) and (X, m') are two abelian variety structures on X with identity elements e and e' respectively. Then m and m' differ only by translation.

Proof:

Let $+$, $-$, and translation all denote operations with respect to m . Consider the morphism $(m - m') : X \times X \rightarrow X$. We have $(m - m')(X \times \{e'\}) = e' = (m - m')(\{e'\} \times X)$. By Corollary 13, $(m - m')(X \times Y) = e'$, i.e. $m = m' + e'$.

BIBLIOGRAPHY

- [1] FULTON, William. *Algebraic Curves*. W. A. Benjamin, Inc., New York (1969).
- [2] MUMFORD, David. *Abelian Varieties*. Oxford University Press, London (1970).
- [3] ——— *Introduction to Algebraic Geometry*. Mimeographed notes, Harvard University.
- [4] SERRE, Jean-Pierre. *Groupes Algébriques et Corps de Classes*. Hermann, Paris (1959).

(Reçu le 22 janvier 1973)

Loren D. Olson

University of Oslo
 Institute of Mathematics
 Blindern Postboks 1053
 Oslo 3, Norvège

vide-leer-empty