Zeitschrift: L'Enseignement Mathématique
Band: 20 (1974)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: CONSTRUCTION OF GAUSS
Kapitel: 1. Introduction
Autor: Barnes, C. W.
DOI: https://doi.org/10.5169/seals-46891

A CONSTRUCTION OF GAUSS

by C. W. BARNES

1. INTRODUCTION

Every prime of the form $4n + 1$ can be expressed uniquely as the sum of two squares. Suppose $p = x^2 + y^2$ where p is a prime of the form $4n + 1$. A construction for x and y was given by Legendre [8] in terms of the continued fraction for $\sqrt{p}$. In [1] we gave a new construction for x and y, again using the continued fraction for $\sqrt{p}$. A summary of the various constructions is given in Davenport [5], pages 120-123.

Gauss [6] remarked that if $p = 4n + 1$, and if α and β are defined by $\beta \equiv \frac{(2n)!}{2(n!)^2} \pmod{p}$, $\alpha \equiv (2n)!\,\beta \pmod{p}$, where $|\alpha| < \frac{p}{2}$, $|\beta| < \frac{p}{2}$ then $p = \alpha^2 + \beta^2$; a particularly simple construction to state. Proofs of the construction of Gauss were given by Cauchy [4], page 414, and Jacobsthal [7]; however, neither of them is simple.

In the present note we give a simple proof of the construction of Gauss based on the method in [1].

2. CONTINUED FRACTIONS

We continue with the notation in [1]. The results we need can be found in Perron [9]. We denote the simple continued fraction

$$a_0 + \cfrac{1}{a_1 + \cfrac{1}{a_2 + \cfrac{\ddots}{\ddots + \cfrac{1}{a_n}}}} \tag{1}$$