1. Introduction

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ON DIOPHANTINE EQUATIONS OF THE FORM $x^2 + D = p^k$

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1. INTRODUCTION

Numerous authors have proved the following theorem:

Theorem 1.1. There are only finitely many solutions of the equation

$$x^2 + D = r^k, \tag{1.1}$$

D, r positive fixed integers; $k = 1, 2, 3, \ldots$.

This has been shown mainly through the theorem of Thue [16] or Landau-Ostrowski [6] which states

Theorem 1.2. The equation

$$ax^2 + bx + c = dy^n \tag{1.2}$$

with n (fixed) $\geqslant 3$, $ad \neq 0$, $b^2 - 4ac \neq 0$ (all letters denoting rational integers) has at most a finite number of solutions.

Proof of Theorem 1.1. Let $a = 1$, $b = 0$, $c = D$. Then equation (1.2) can be transformed into

$$x^2 + D = dy^n. \tag{1.3}$$

Now let $y = r^\nu$, $\nu = [k/3]$, and $d = 1, r, r^2$ if $k \equiv 0, 1, 2 \pmod 3$, respectively. Then equation (1.3) becomes three equations of the type (1.2) when $n = 3$. ∇

Mordell [9] proved Theorem 1.2 when $n = 3$. This is obviously sufficient for our purposes.

Henceforth, r will be a prime p and $D \equiv 3 \pmod 4$ [unless explicitly stated otherwise]. Under the conditions that r be a prime, Apéry proved [1], [2] that equation (1.1) has at most two solutions unless $D = 7$ and $p = 2$.

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