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11. Class Field Theory

Some of the origins of the cohomology of groups—specifically, the factor sets for crossed product algebras—came from class field theory. Hence it is not surprising that one of the principle uses of this cohomology lies back in class field theory. Possibilities of this sort were in the minds of Eilenberg and Mac Lane when they wrote a paper applying cohomology of groups along the lines of the earlier Teichmüller work [1940] on 3-cocycles. Mac Lane also recalls that Artin (about 1948) pointed out in conversations that the cohomology of groups should have use in class field theory. Hochschild [1950] and Hochschild and Nakayama [1952] showed how the Brauer group arguments of class field theory could be replaced by cohomological arguments. In 1952, Tate proved that the homology and cohomology groups for a finite group G could be suitably combined in a single long exact sequence. He used this sequence, together with properties of transfer and restriction, to give an elegant reformulation of class field theory. It is still today one of the effective approaches to this subject—as presented, for example, in the recent book of Iyanaga and Iyanaga [1975].

12. Homological Algebra

The discovery of the cohomology of groups was an essential part of the development of homological algebra. This subject, as organized by Cartan and Eilenberg, provides a unified way of accounting for a variety of new functors, starting with the cohomology of groups. Such are:

$H^n(G, A)$, the cohomology of a group G, with coefficients in a left G-module A;

$H_n(G, A)$, the homology of a group G, with coefficients in a right G module A;

$H^n(\Lambda, A)$, the (Hochschild) cohomology of an algebra Λ, with coefficients in a Λ-bimodule A;

$H^n(g, C)$, the cohomology of the Lie algebra g, with coefficient in a g-module C;

Ext (A, B), the group of abelian group extensions of the abelian group B by the abelian group A;

Tor (A, B), the torsion product of the abelian groups A and B.