

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 25 (1979)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** INTEGRAL REPRESENTATION THEOREMS VIA BANACH ALGEBRAS  
**Autor:** Maltese, George  
**Kapitel:** 1. Introduction  
**DOI:** <https://doi.org/10.5169/seals-50383>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

**Download PDF:** 06.02.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

# INTEGRAL REPRESENTATION THEOREMS VIA BANACH ALGEBRAS

by George MALTESE

## 1. INTRODUCTION

Many classical integral representation theorems of analysis can be obtained as special cases of the Choquet Representation Theorem [6], [7], [14] or the Krein-Milman Theorem. The procedure involves the definition of a suitable convex compact set in some locally convex space and an explicit description of the extreme points of this set. The latter is often a non-trivial task, therefore it seems appropriate to develop alternative methods which are general enough to yield a class of integral representation theorems. In many situations in which an integral representation formula is sought, there is a natural commutative Banach algebra inherent in the background. For example in the case of Bochner's theorem for positive definite functions on a locally compact abelian group  $G$ , the natural Banach algebra is the convolution algebra  $L^1(G)$ . In the case of the Schoenberg-Eberlein theorem for Fourier-Stieltjes transforms on locally compact abelian groups, the Banach algebra is again the convolution algebra. In the case of the Spectral Theorem for a normal operator  $T$  on a Hilbert space  $\mathcal{H}$ , the natural Banach algebra is the closed commutative  $*$  algebra generated by  $T$  and the identity operator.

In this paper we show that the above mentioned theorems are all special cases of a general result (Theorem 1) on the integral representation of certain linear forms defined on commutative Banach algebras. Specialization of Theorem 1 to symmetric Banach algebras yields a generalized version (Theorem 2) of a result of Raikov [10] for positive functionals on such algebras.

The proof of Theorem 1 is straight forward and its version for positive functionals on involution algebras is classical [11]. The main point here is the relative ease of application of Theorem 1 to a variety of situations.