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Application 4 (The spectral theorem for normal operators). Let \mathcal{H} be a Hilbert space and let $\mathcal{L}(\mathcal{H})$ denote the Banach algebra of all bounded linear operators on \mathcal{H} . Consider a subalgebra $A \subset \mathcal{L}(\mathcal{H})$ with the following properties:

- (i) A is commutative;
- (ii) A is closed;
- (iii) If $T \in A$ then $T^* \in A$;
- (iv) The identity operator belongs to A .

Let Δ denote the maximal ideal space of A . Since each $T \in A$ is normal it follows that $\|T\| = \|\hat{T}\|_\infty$ for every $T \in A$.

For each pair of vectors $\xi, \eta \in \mathcal{H}$ define a mapping $L_{\xi, \eta} : A \rightarrow \mathbb{C}$ by

$$L_{\xi, \eta}(T) = (T\xi, \eta)$$

then we have

$$|L_{\xi, \eta}(T)| \leq \|T\| \cdot \|\xi\| \|\eta\| = \|\xi\| \|\eta\| \cdot \|\hat{T}\|_\infty$$

Therefore by Theorem 1 there exists a bounded complex Radon measure $\mu_{\xi, \eta}$ on Δ such that $\|\mu_{\xi, \eta}\| \leq \|\xi\| \cdot \|\eta\|$ and

$$L_{\xi, \eta}(T) = \int_{\Delta} \hat{T} d\mu_{\xi, \eta} \quad \text{for every } T \in A.$$

An application of the Gelfand-Neumark theorem establishes the uniqueness of the measure. The usual construction of a unique resolution of the identity on the Borel sets of Δ can be made based on this formula. A specialization of this formula to a single normal operator leads to the classical spectral theorem. We shall not give the details here since many excellent accounts exist (c.f. Berberian [1], [2], Segal-Kunze [18]). An especially lucid presentation is given in Rudin [16].

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