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where $N = \ker(\pi_1(X) \rightarrow \pi_1(Y))$. By (6.6) the map β_i is i -connected since the fiber of the two vertical arrows is $A(P_n \tilde{X})_N$. Now by (5.4) we see that α_i is simple for $k > i$.

For two decompositions (X'_i) and (X''_i) of $f : X \rightarrow Y$ satisfying the above conditions, we have $P_i X'_i = P_i X''_i$ and both X'_i and X''_i map into X_i , constructed above, such that the resulting diagrams are homotopy commutative. The connectivity of the β_i and (5.1) shows that these maps are all homotopy equivalences. This proves the theorem.

(6.8) *Remarks.* This theorem (6.7) coincides with the Dror results for Y a point [D1, Theorem 1.3] and $Y = S^n$ [D2]. An interesting problem is to describe the i th stage X_i in terms of invariants of X_{i-1} as in [D1] and [D2]. (See the footnote in the introduction.)

APPENDIX — SIMPLICITY PROPERTIES OF FIBERS

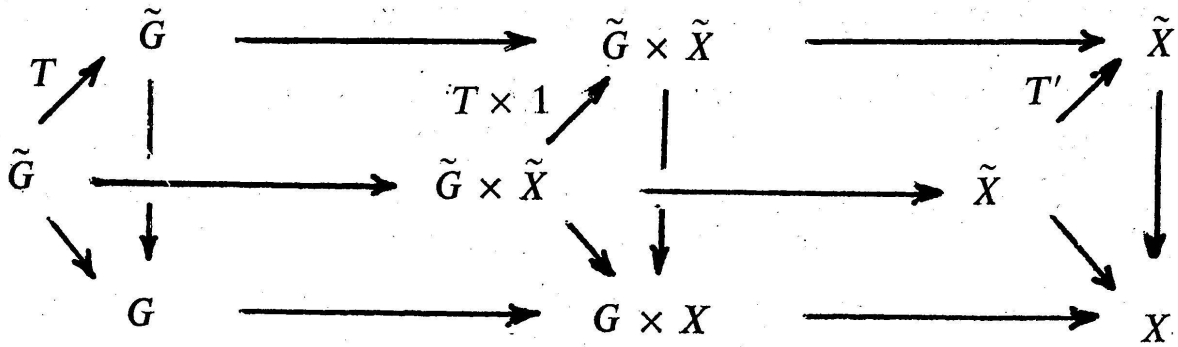
In the proof of (5.4) we used the fact that for a fibration $F \rightarrow E \xrightarrow{f} B$ the action of $\pi_1(F)$ on $\text{Im}(\partial : \pi_{k+1}(B) \rightarrow \pi_k(F))$ is trivial. This assertion does not seem to be in the literature so we include a proof here.

We extend the mapping sequence of the fibration f to $\Omega B \rightarrow F \rightarrow E \xrightarrow{f} B$ and study F as the total space of a principal fibration with fibre the H -space ΩB . If G is an H -space, then $\pi_1(G)$ acts trivially on $\pi_*(G)$ because the covering transformations $\tilde{G} \rightarrow G$ on the universal covering \tilde{G} of G are homotopic to the identity. This is proved by lifting a loop to a path in \tilde{G} and using the H -space structure on \tilde{G} to deform the identity along this path to the covering transformation defined by the homotopy class of the loop. Recall that a principal fibration is induced from $G \rightarrow E_G \rightarrow B_G$ up to fibre homotopy equivalence.

(A.1) PROPOSITION. *Let $G \rightarrow X \xrightarrow{\pi} Y$ be a principal fibration with fibre G acting on X . Then we have:*

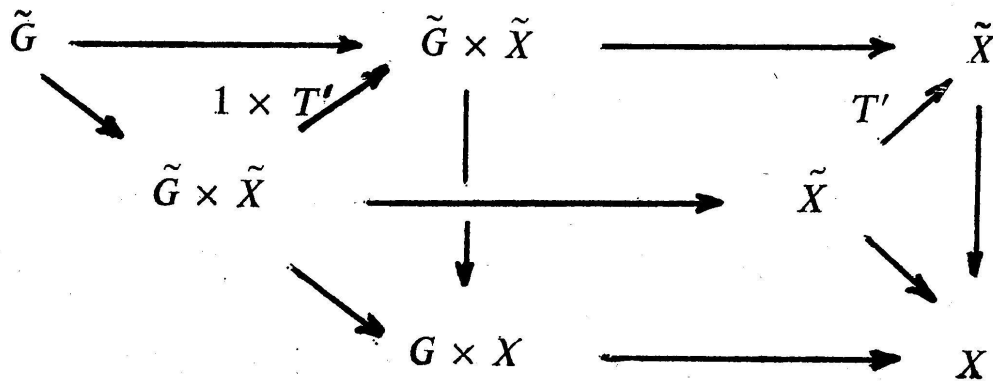
- (a) $\text{im}(\pi_1(G) \rightarrow \pi_1(X))$ acts trivially on $\pi_*(X)$, and
- (b) $\pi_1(X)$ acts trivially on $\text{im}(\pi_*(G) \rightarrow \pi_*(X))$.

Proof. For (a) we have the following commutative diagram induced by a covering transformation $T : \tilde{G} \rightarrow \tilde{G}$.



The covering transformation T defines T' , and since T is homotopic to the identity so is T' . This proves (a).

For (b) we use the following commutative diagram where T' is any covering transformation of \tilde{X} .



Now the inclusion $i : \tilde{G} \rightarrow \tilde{X}$ is the composite of the first horizontal row, and $T'i$ and i are homotopic by $i_t(g) = g \cdot \tilde{\alpha}(t)$ where $g \in G$ and α is a lifting of the loop α corresponding to the covering transformation T' . This proves the proposition.

For a general fibration $f : E \rightarrow B$ with fibre F the mapping sequence $\Omega B \rightarrow F \rightarrow E \rightarrow B$ allows us to deduce the next proposition from the previous one.

(A.2) PROPOSITION. *Let $f : E \rightarrow B$ be a fibration with fibre $F \rightarrow E$. Then we have :*

- (a) $\text{im}(\pi_2(B) \rightarrow \pi_1(F))$ acts trivially on $\pi_*(F)$, and
- (b) $\pi_1(F)$ acts trivially on $\text{im}(\partial : \pi_{i+1}(B) \rightarrow \pi_i(F))$.