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*Characterization B5.* The quadratic form of  $f$  is negative definite.

The equivalence of Characterizations B1 and B5 is proved in [Tjurina 1]. By explicit computation the quadratic forms of the germs in Table 2a are shown to be negative definite, and those of Table 2b are shown to be negative semi-definite. (In fact, the quadratic form of a germ in Table 2a is isomorphic to the intersection pairing of its minimal resolution, and the quadratic form of a germ of type  $\tilde{E}_k$  in Table 2b is isomorphic to the quadratic form of  $E_k$  plus a two-dimensional zero form.) The result then follows from Proposition 10.1 and Lemma 12.1. When  $n = 2$ , the Milnor fiber  $F$  is in fact diffeomorphic to the minimal resolution  $M$  of  $f^{-1}(0)$ , since the singularity of  $f^{-1}(0)$  is an absolutely isolated double point [Brieskorn 1, Theorem 4; Tjurina 1, Lemma 1].

When  $n = 2$ , the equivalence of Characterizations A2 and B5 follows from the following result [Durfee 2, Proposition 3.1].

**THEOREM 12.2.** *Twice the geometric genus  $p$  of  $f^{-1}(0)$  equals the number of positive plus the number of zero diagonal elements in a diagonalization of the intersection pairing over the real numbers.*

The classification of germs according to signature of the quadratic form has been extended in [Arnold 3]; see also [Durfee 2, Proposition 3.3].

### 13. NEARBY MORSE FUNCTIONS

A *deformation* of a germ  $f \in \mathcal{F}$  is a germ  $g: \mathbf{C}^{n+1} \times \mathbf{C} \rightarrow \mathbf{C}$  with  $g(z, 0) = f(z)$ . Choose  $\varepsilon$  and  $\delta$  for  $f$  as in §11. Then choose  $\eta > 0$  such that for all  $|t| < \eta$  and  $|\delta'| \leq \delta$ , the set  $\{z \in \mathbf{C}^{n+1}: g(z, t) = \delta'\}$  intersects  $S_\varepsilon^{2n+1}$  transversally and the critical values of  $g(z, t)$  for fixed  $t$  are less than  $\delta$  in absolute value. A germ  $\bar{f}$  is a *nearby Morse function* to  $f$  if  $\bar{f}$  has only non-degenerate critical points in  $D_\varepsilon^{2n+2}$  and there is a deformation  $g$  and a  $t_0$  with  $|t_0| < \eta$  such that  $\bar{f}(z) = g(z, t_0)$ .

*Characterization B6.* There is a nearby Morse function to  $f$  with one or two critical values.

In fact, the nearby Morse function has one critical value if and only if  $f$  is right equivalent to  $A_2$ , since the quadratic form diagram is connected (§14). This surprising characterization is in [A'Campo 2II], where it is shown that Characterization B1 implies B6, and B6 implies B7 below.