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#### 14. VANISHING CYCLES

Let  $f$  be a germ in  $\mathcal{F}$ , and let  $\bar{f}$  be a nearby Morse function with  $\mu$  distinct critical values  $t_1, \dots, t_\mu$  in the disk  $D_\delta^2$  of radius  $\delta$  about 0 in  $\mathbb{C}$ . A path  $\alpha_i$  in  $D_\delta^2 - \{t_1, \dots, t_\mu\}$  from  $\delta$  to  $t_i$  determines (up to sign) a *vanishing cycle*  $\delta_i$  in  $H_n(F)$ . The self-intersection  $(\delta_i, \delta_i)$  is  $2(-1)^{n/2}$  or 0 according as  $n$  is even or odd. Choose paths  $\alpha_1, \dots, \alpha_\mu$  in  $D_\delta^2 - \{t_1, \dots, t_\mu\}$  from  $\delta$  to  $t_1, \dots, t_\mu$  respectively, such that the union of the images of the paths is a deformation retract of  $D_\delta^2$ ; then the corresponding vanishing cycles  $\delta_1, \dots, \delta_\mu$  are a basis of  $H_n(F)$  [Brieskorn 4, Appendix]. The basis  $\delta_1, \dots, \delta_\mu$  is called an *ordered* (or *distinguished*) *basis of vanishing cycles* if  $t_1, \dots, t_\mu$  are ordered so that the loop going once counter-clockwise around the boundary of  $D_\delta^2$  is homotopic in  $\pi_1(D_\delta^2 - \{t_1, \dots, t_\mu\}, \delta)$  to the product  $\beta_1 * \dots * \beta_\mu$ , where  $\beta_i$  is the loop going out  $\alpha_i$  almost to  $t_i$ , around  $t_i$  counter-clockwise, and back along  $\alpha_i$ . References for this are [Gabrielov 1, Lamotke, Durfee 1].

Choose an ordered basis of vanishing cycles  $\delta_1, \dots, \delta_\mu$  for the intersection pairing  $(,)$  of  $f(z_0, \dots, z_n) + z_{n+1}^2 + \dots + z_m^2$ , where  $m \equiv 2 \pmod{4}$ . The *quadratic form diagram* of  $f$  with respect to the basis  $\delta_1, \dots, \delta_\mu$  has vertices  $v_1, \dots, v_\mu$  and edges from  $v_i$  to  $v_j$  if  $(\delta_i, \delta_j) \neq 0$ , weighted by  $(\delta_i, \delta_j)$  if  $(\delta_i, \delta_j) \neq 1$ . This diagram is connected [Lazzeri; Gabrielov 2]. It determines all the topological information in the singularity if  $n \neq 2$  [Durfee 1]. There are a number of methods of computing these diagrams [A'Campo 2I; Gabrielov 3; Gusein-Zade]. The quadratic form diagrams of the germs of Table 2 are listed in column 5. Lemma 12.1 can be strengthened to show that if  $f$  topologically degenerates to  $g$ , then some quadratic form diagram for  $f$  is a subdiagram of some quadratic form diagram for  $g$  [Siersma, p. 82].

*Characterization B7.* There is an ordered basis of vanishing cycles for  $f$  such that the quadratic form diagram is a (weighted) tree.

It is shown in [A'Campo 2II] that Characterizations B1 and B7 are equivalent. In fact, the quadratic form diagrams for the germs in Table 2a are the same as the graph of their minimal resolutions (column 3 of Table 1).

#### 15. THE MONODROMY GROUP

Let  $f$  be a germ in  $\mathcal{F}$ , and as above choose an ordered basis  $\delta_1, \dots, \delta_\mu$  of vanishing cycles for  $H_m(F)$ , where  $F$  is the Milnor fiber of

$$f(z_0, \dots, z_n) + z_{n+1}^2 + \dots + z_m^2$$