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Autor: Nachbin, Leopoldo
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WHY HOLOMORPHY IN INFINITE DIMENSIONS?

by Leopoldo NACHBIN

> Ein Mathematiker, der nicht etwas Poet ist, wird nie ein vollkommener Mathematiker
>
> KARL WEIERSTRASS

1. INTRODUCTION

The study of holomorphic functions in infinite dimensions is an objective as old in Mathematics as Functional Analysis, and as the idea of systems with an infinite number of degrees of freedom in Mechanics. It dates back to the end of the last century. The simple language of normed spaces and of topological vector spaces became a routine, as a suitable form of linear algebra in infinite dimensions to be used in Analysis, Geometry and applications. Thereafter, the theory of holomorphic mappings in infinite dimensions was properly developed as a confluence of ideas and methods originating mostly from several complex variables, manifold theory and Functional Analysis. Independently of that, users of sophisticated mathematical methods in applications have employed and furthered holomorphy in infinite dimensions, in fields such as Mathematical Physics and Electrical Engineering. The present expository article was written by aiming at the non-specialists, more exactly, at the non-mathematicians. We will use Weierstrass' definition as a model for the general case.

2. SOME CLASSICAL MOTIVATIONS

EXAMPLE 1: SPECTRAL THEORY. If $Z: E \to E$ is a linear operator on the complex vector space E of finite dimension $n = 1, 2, \ldots$, the homogeneous linear equation $Z(x) = \lambda x$ has at least some solution $x \in E$, $x \neq 0$ for at least some $\lambda \in \mathbf{C}$. Equivalently, there is at least some $\lambda \in \mathbf{C}$ such that $\lambda I - Z$ is not invertible in the algebra $\mathscr{L}(E; E)$ of all linear operators on E, where I is the identity mapping of E; the set of all such λ has at most n elements. This fact is proved by noticing that $\lambda I - Z$ is not invertible if and only if,