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**Autor:** Ballico, Edoardo  
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## ON COUSIN-I COMPLEX SPACES

by Edoardo BALLICO <sup>1)</sup>

### SUMMARY

We study the properties of a complex space such that every closed analytic subspace has the 1-Cousin property. Under mild hypotheses we prove that  $X$  is a Stein space.

G. Berg in [2] proved very easily the following result: Let  $X$  be an open subset of a two-dimensional Stein space  $Y$ . If in  $X$  every 1-Cousin problem is solvable, then  $X$  is a domain of holomorphy in  $Y$ . It is not difficult to modify his proof and obtain that, under the stated assumption,  $X$  is always a Stein space.

In this note we want to consider this problem also for higher dimensional complex spaces. The proofs are always by induction on the dimension; they are similar to the proofs in the two dimensional case. This is also a partial generalization of my previous [1], theorem 2. We say that a complex space  $X$  is Cousin-I or has the 1-Cousin property if every 1-Cousin problem on  $X$  is solvable. Any Stein space is a Cousin-I space. We denote by  $O(X)$  the algebra of holomorphic functions on a complex space  $X$ . For each  $f \in O(X)$  we put

$$\{x \in X : f(x) = 0\} =: V(f),$$

an analytic subset of  $X$ . In this paper we consider for simplicity only complex spaces with bounded Zariski tangent dimension.

LEMMA 1. *Let  $X$  be a complex space with the 1-Cousin property and  $f \in O(X)$  such that for each  $x \in X$  the germ of  $f$  around  $x$  is a non-zero divisor in  $\mathcal{O}_{X,x}$ . We put*

$$Z := (V(f), \mathcal{O}_Z)$$

with

$$\mathcal{O}_Z := \mathcal{O}_X / f \mathcal{O}_{X|V(f)}.$$

Then the natural map  $p: O(X) \rightarrow O(Z)$  is surjective.

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<sup>1)</sup> The author is member of G.N.S.A.G.A. of C.N.R. (Italy).