Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 27 (1981)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON COUSIN-I COMPLEX SPACES
Autor: Ballico, Edoardo
Kurzfassung: Summary
DOI: https://doi.org/10.5169/seals-51755

ON COUSIN-I COMPLEX SPACES

by Edoardo BALLICO [1])

SUMMARY

We study the properties of a complex space such that every closed analytic subspace has the 1-Cousin property. Under mild hypotheses we prove that X is a Stein space.

G. Berg in [2] proved very easily the following result: Let X be an open subset of a two-dimensional Stein space Y. If in X every 1-Cousin problem is solvable, then X is a domain of holomorphy in Y. It is not difficult to modify his proof and obtain that, under the stated assumption, X is always a Stein space.

In this note we want to consider this problem also for higher dimensional complex spaces. The proofs are always by induction on the dimension; they are similar to the proofs in the two dimensional case. This is also a partial generalization of my previous [1], theorem 2. We say that a complex space X is Cousin-I or has the 1-Cousin property if every 1-Cousin problem on X is solvable. Any Stein space is a Cousin-I space. We denote by $O(X)$ the algebra of holomorphic functions on a complex space X. For each $f \in O(X)$ we put

$$\{x \in X : f(x) = 0\} =: V(f),$$

an analytic subset of X. In this paper we consider for simplicity only complex spaces with bounded Zariski tangent dimension.

LEMMA 1. *Let* X *be a complex space with the* 1-*Cousin property and* $f \in O(X)$ *such that for each* $x \in X$ *the germ of* f *around* x *is a non-zero divisor in* $\mathcal{O}_{X,x}$. *We put*

$$Z := (V(f), \mathcal{O}_Z)$$

with

$$\mathcal{O}_Z := \mathcal{O}_X / f\, \mathcal{O}_{X|V(f)}\,.$$

Then the natural map $p : O(X) \to O(Z)$ *is surjective.*

[1]) The author is member of G.N.S.A.G.A. of C.N.R. (Italy).