$$S(a_1, \ldots, a_{k-1}, a_k; q) = S(a_1, \ldots, a_{k-1}, a_{k_1}; q_1)\, S(a_1, \ldots, a_{k-1}, a_{k_2}; q_2) .$$

Thus by the inductive assumption

$$\begin{aligned} &| S(a_1, \ldots, a_{k-1}, a_k; q) | \\ &\leqslant k^{v(q_1)} (q_1)^{\frac{k-1}{2}} (a_1, a_{k_1}, q_1)^{1/2} \ldots (a_{k-1}, a_{k_1}, q_1)^{1/2} \\ &\cdot k^{v(q_2)} (q_2)^{\frac{k-1}{2}} (a_1, a_{k_2}, q_2)^{1/2} \ldots (a_{k-1}, a_{k_2}, q_2)^{1/2}. \end{aligned}$$

Since it is easily seen

$$\begin{aligned} (a_1, a_{k_1}, q_1)(a_1, a_{k_2}, q_2) &= (a_1, a_k, q) \\ &\vdots \\ (a_{k-1}, a_{k_1}, q_1)(a_{k-1}, a_{k_2}, q_2) &= (a_{k-1}, a_k, q) \end{aligned}$$

the theorem is proved.

Theorem 2 is proved similarly.

Note. By symmetry, the $(a_1, a_k, q)^{1/2} \ldots (a_{k-1}, a_k, q)^{1/2}$ term in Theorems 1 and 2 may be replaced by

$$\begin{aligned} \min \{ &(a_1, a_k, q)^{1/2} (a_2, a_k, q)^{1/2} \ldots (a_{k-1}, a_k, q)^{1/2} , \\ &(a_1, a_{k-1}, q)^{1/2} (a_2, a_{k-1}, q)^{1/2} \ldots (a_k, a_{k-1}, q)^{1/2} , \\ &\vdots \\ &(a_2, a_1, q)^{1/2} (a_3, a_1, q)^{1/2} \ldots (a_k, a_1, q)^{1/2} \} . \end{aligned}$$

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(Reçu le 14 juin 1980)

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