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Artikel: SPECKER'S MATHEMATICAL WORK FROM 1949 TO 1979
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VIII. Logic and quantum mechanics: 14 (and 25), 19, 20, 21.

IX. Algorithms: 23, 25, 27, 38, 30, 31.

X. General: 7, 32.

Roughly speaking, over the last thirty years or so, the first 15 years are devoted to the logician's traditional concern with set theory, analysis, and arithmetic (with the work on topology as a prelude supplying a solid training in ordinary mathematics), while the second 15 years are devoted to the logic of quantum mechanics and the study of algorithms. Rather than enumerating the main theorems in the different articles, I shall select more or less arbitrarily to remark on a few of the articles which are representative and not too unfamiliar to me.

From doctorate to professorship

Specker received his doctorate of mathematics in 1948 with his Promotionsarbeit in topology (published as 1949*a*, 1 in the above list). He completed his Habilitationsschrift in axiomatic set theory in 1951, which was later published as 1954*b* and 1957*a* (8 and 9 in the above list). In these studies he worked closely with H. Hopf and Bernays. Later on October 16, 1976 he gave a lecture (29 in the above list) in München on the occasion of the Ehrenpromotion of Bernays in which he traced the development of axiomatic set theory with special attention to the contributions by Bernays. In his Habilitationsschrift, Specker proves the independence of the axiom of foundation, gives a new proof of the independence of the axiom of choice from the axioms of set theory minus the axiom of foundation, studies several alternatives to the axiom of choice, and sharpens results dealing with the relation between the axiom of choice and the generalized continuum hypothesis.

In 1954 Specker gave his Antrittsvorlesung at the E.T.H., in which he discusses the conceptual foundation of set theory (published as 7 in the above list). This is probably the only publication by Specker which would ordinarily be classified as "philosophical" in the specialized academic sense of the word and, at least in his published work, Specker has so far not returned to philosophical considerations which do not directly suggest some mathematical problems. In his general paper on Bernays (32), he notes that about half of Bernays' papers may be classified as philosophical: this difference between them may be partly due to the different historical periods in which they live.

Enjoyment of interaction

Among Specker's publications, several papers seem to have been stimulated primarily by the enjoyment of personal interaction. Thus the paper 1949*c* dealt with a problem of Sikorski who was visiting Zürich then, while the paper 1964 continued the study to more elaborate cases (3 and 18 in the above list). The papers 1957*b* and 1961*b* (10 and 16 in the above list) seem to belong to the class of papers which are provoked by the infinite supply of problems from Erdös. Paper 11 answers a problem raised by Mostowski. A most obviously playful paper is 1978*b* (30) which gives, for the recognition problem, the generating problem, and the counting problem of the partition of finite sets, algorithms programmable on the "toy" computer HP-25.

Several of these papers contain clever constructions which stimulate extensions and generalizations. For example, the paper 10 gives the Specker graph which shows:

$$\omega^3 \rightarrow (2, \omega^3)^2 \text{ and } \omega^3 \not\rightarrow (3, \omega^3)^2 .$$

This leads to the function $f(n)$ such that $f(n) < \omega$,

$$\omega^n \rightarrow (f(n)-1, \omega^3)^2 \text{ and } \omega^n \not\rightarrow (f(n), \omega^3)^2 .$$

Eva Nosal many years later showed that $f(n) = 2^{n-2} + 1$ for $n \geqslant 3$, *J. London Math. Soc. (2), 8* (1974), 306-310.

Topology and recursive analysis

It is interesting to observe that Specker's early papers of 1949 and 1950 have continued to interest mathematicians over the years. For example, the paper 2 gives a bounded increasing recursive sequence of rational numbers that does not converge to a recursive real number. In a recent paper by M. I. Kanovič, such sequences are called Specker sequences, and the complexity of "limit candidates" for a Specker sequence is studied with the result that the larger the complexity of the candidate, the closer it is to