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THE RIEMANN-ROCH THEOREM FOR COMPACT RIEMANN SURFACES

by R. R. SIMHA

§ 1. INTRODUCTION

The aim of this article is to present a sheaf-theoretic proof of the Riemann-Roch theorem (including Serre duality) for vector bundles on compact Riemann surfaces. The basic assumption will be the finite dimensionality of cohomology vector spaces; no further potential theory will be used. Thus the proof will work (with trivial modifications) in the algebraic case also (over an algebraically closed field of any characteristic). The possibly new contribution of the article is a simple direct proof of the fact that the degree of the canonical divisor is $2g - 2$, where $g = \dim H^1(X, \mathcal{O})$.

We now give an outline of the contents. The rather long Section 2 gives the necessary definitions and sheaf-theoretic results, and the consequences of the finite dimensionality theorem which are needed later. Section 3 gives the preliminary form of the Riemann-Roch theorem. The identity $\deg K = 2g - 2$ is proved in Section 4. Serre duality and the final form of the Riemann-Roch theorem are proved in Section 5.

Our exposition borrows freely from those of Serre [5] and Mumford [4]. We should also mention the proof of the Riemann-Roch theorem given in Grauert-Remmert [1] (Ch. VII).

I thank the referee for his careful reading of the manuscript, which has eliminated many errors.

§ 2. LINE BUNDLES AND VECTOR BUNDLES. SHEAF THEORETIC PRELIMINARIES

In all that follows, X will denote a compact Riemann surface, i.e. a connected compact complex manifold of complex dimension 1; $\mathcal{O} = \mathcal{O}_X$ will denote its structure sheaf, i.e. the sheaf of germs of holomorphic functions on X.