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Autor: Simha, R. R.

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we need only show that, if $\sigma \in H^0(X, \mathcal{L})$ is such that $\zeta(\sigma \otimes \gamma) = 0$ for all $\gamma \in H^1(X, K \otimes \mathcal{L}^{-1})$, then $\sigma \equiv 0$. Now choose any $P \in X$, and a coordinate neighbourhood (U, z) of P centred at P such that $\mathcal{L}|_U \approx \mathcal{O}_U$. Then the covering $\mathfrak{U} = \{U, X - P\}$ is a Leray covering for \mathcal{L}, K and $K \otimes \mathcal{L}^{-1}$ ((3.7)). The $z^n dz, n \in \mathbf{Z}$, can all be regarded as elements of $Z^1(\mathfrak{U}, K \otimes \mathcal{L}^{-1})$; let γ_n denote their images in $H^1(X, K \otimes \mathcal{L}^{-1})$. Then clearly $\rho(\sigma \otimes \gamma_n) = 0$ for all n implies that all the coefficients of the Taylor expansion of σ at P with respect to vanish, hence $\sigma \equiv 0$, q.e.d.

(5.9) SERRE DUALITY FOR VECTOR BUNDLES. For any vector bundle \mathcal{V} on X , let $\mathcal{V}^* = \text{Hom } \mathcal{O}_X(\mathcal{V}, \mathcal{O}_X)$. Then the natural pairing

$$\zeta : H^0(X, \mathcal{V}) \times H^1(X, K \otimes \mathcal{V}^*) \rightarrow H^1(X, K) \xrightarrow{\text{res}} \mathbf{C}$$

is non-degenerate.

Proof: Arguing as in the proof of (5.8) we see that the map $H^0(X, \mathcal{V}) \rightarrow (H^1(X, K \otimes \mathcal{V}^*))^*$ induced by ζ is injective, hence $h^0(X, \mathcal{V}) \leq h^1(X, K \otimes \mathcal{V}^*)$. Replacing \mathcal{V} by $K \otimes \mathcal{V}^*$, we also get $h^0(K \otimes \mathcal{V}^*) \leq h^1(\mathcal{V})$. But, by induction on rank \mathcal{V} , we easily deduce from (5.3) that $\chi(K \otimes \mathcal{V}^*) = -\chi(\mathcal{V})$, hence $h^0(X, \mathcal{V}) = h^1(X, K \otimes \mathcal{V}^*)$. Thus ζ is non-degenerate as before.

REFERENCES

- [1] GRAUERT, H. and R. REMMERT. *Theory of Stein Spaces*. Springer-Verlag, 1979.
- [2] GUNNING, R. C. *Lectures on Riemann surfaces*. Princeton University Press.
- [3] GUNNING, R. C. and H. ROSSI. *Analytic Functions of Several Complex Variables*. Prentice Hall, 1965.
- [4] MUMFORD, D. *Algebraic Geometry I: Complex Projective Varieties*. Springer-Verlag, 1976.
- [5] SERRE, J.-P. *Groupes Algébriques et Corps de Classes*. Hermann, 1959.

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R. R. Simha

School of Mathematics
Tata Institute of Fundamental Research
Bombay 400 005
India