

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 27 (1981)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: IDENTITIES FOR PRODUCTS OF GAUSS SUMS OVER FINITE FIELDS
Autor: Evans, Ronald J.
Kapitel: 3. Theorems of Stickelberger and Davenport-Hasse
DOI: <https://doi.org/10.5169/seals-51748>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. [Siehe Rechtliche Hinweise.](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. [Voir Informations légales.](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. [See Legal notice.](#)

Download PDF: 06.02.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

3. THEOREMS OF STICKELBERGER AND DAVENPORT-HASSE

We will make use of the following three classical formulas. First [3, (0.8)],

$$(6) \quad G_{f_m} \left(\chi \frac{q^m - 1}{q - 1} \right) = G_f(\chi)^m,$$

where χ is a character on $GF(q^m)$. Next [3, (0.9)],

$$(7) \quad 1 = \frac{\chi^l(l)}{G_f(\chi^l)} \prod_{j=0}^{l-1} \frac{G_f(\chi\psi^j)}{G_f(\psi^j)},$$

where χ, ψ are characters on $GF(q)$ and ψ has order l . Finally [8], [5, p. 25]

$$(8) \quad \frac{G_f(\chi^\alpha)}{(\zeta - 1)^{s(\alpha)}} \equiv \frac{1}{\gamma(\alpha)} \equiv \frac{(\zeta - 1)^{\alpha - s(\alpha)}}{\alpha!} \pmod{P},$$

where α is an integer, $0 \leq \alpha < q - 1$; $s(\alpha)$ denotes the sum of the p -adic digits of α ; $\gamma(\alpha)$ denotes the product of the factorials of the p -adic digits of α ; P is a prime ideal above p in the ring $\mathcal{O} = \mathbb{Z}[\omega]$, where $\omega = \exp(2\pi i/p(q-1))$; and χ is the character on $\mathcal{O}/P \approx GF(q)$ of order $q - 1$ which maps the coset $\omega + P$ to $\bar{\omega}$.

4. PROOF OF (2)

Let η denote the right side of (2). We must show that $\eta = 1$. Let $\delta = \frac{q^n - 1}{q - 1}$, $\theta = w^{k-1}(cw + i_j)$. Using (6), we have

$$\eta^n = \frac{\chi^{ln}(l) G_{f_n}(\chi^\delta)}{G_{f_n}(\chi^{\delta l})} \prod_{j=1}^e \prod_{k=1}^r \prod_{c=1}^{w^{r-k}} \frac{G_{f_n}^n(\chi^\delta \psi^\theta)}{G_{f_n}^n(\psi^\theta)}.$$

Consider a fixed pair j, k . For each $a \in \{1, 2, \dots, n\}$, $G_{f_n}(\psi^\theta) = G_{f_n}(\psi^{\theta q^a})$, so

$$\prod_{c=1}^{w^{r-k}} G_{f_n}(\psi^\theta) = \prod_{c=1}^{w^{r-k}} G_{f_n}(\psi^{w^{k-1}(cw + i_j q^a)}).$$

Similarly,

$$\prod_{c=1}^{w^{r-k}} G_{f_n}(\chi^\delta \psi^\theta) = \prod_{c=1}^{w^{r-k}} G_{f_n}(\chi^\delta \psi^{w^{k-1}(cw + i_j q^a)}).$$