Dupont and Sah show that the volume function and the sharpened Dehn invariant can be incorporated into a single function ρ, as follows. Let

$$\rho(z) = 1 \wedge L(z) - 1 \wedge L(1-z) + l(z) \wedge l(1-z),$$

with values in $\wedge^2\mathbf{C}$, where $l(z) = \log(z)/2\pi i$ and

$$L(z) = \mathscr{L}_2(z)/4\pi^2 = \int_0 l(1-z)dl(z).$$

This expression is certainly well defined in the strip $0 < \mathrm{Re}(z) < 1$, and satisfies $\rho(z) + \rho(1-z) = 0$. If we analytically continue each of its constituent functions in a loop around zero or one, then the expression $\rho(z)$ remains unchanged. Hence ρ is well defined as a mapping from $\mathbf{C} - \{0, 1\}$ to $\wedge^2\mathbf{C}$. They show that ρ also satisfies the symmetry condition (31), the Kubert identity (32), and the cocycle equation (33).

REFERENCES

[1] Artin, E. *The Gamma Function.* Holt, Rinehart and Winston 1964.

[2] Bass, H. Generators and relations for cyclotomic units. *Nagoya Math. J. 27* (1966), 401-407.

[3] Bateman, H., edited by Erdélyi *et al. Higher Transcendental Functions, vol. I.* McGraw-Hill, 1953. (See pp. 24-39.)

[4] Bloch, S. Applications of the dilogarithm function in algebraic K-theory and algebraic geometry. *Proc. Conf. Algebraic Geom. Kyoto Univ. 1977.* (See also "Algebraic K-theory and zeta functions of elliptic curves", *Proc. Int. Congr. Math. Helsinki 1978*, II, 511-515; and "The dilogarithm and extensions of Lie algebras", preprint, U. Chicago 1981.)

[5] Borevich, Z. I. and I. R. Shafarevich. *Number Theory.* Academic Press 1966.

[6] Chudnovsky, G. V. Padé approximations to the generalized hypergeometric functions. I, *J. Math. pures appl. 58* (1979), 445-476.

[7] Ewing, J. Spheres as fixed point sets. *Quart. J. Math., Oxford, 27* (1976), 445-455.

[8] Hilbert, D. Die Theorie der algebraischen Zahlkörper. *Jahresb. D. Math.-Ver. 4* (1897), 175-546. (See §120.)

[9] Hurwitz, A. Einige Eigenschaften der Dirichlet'schen Funktionen $F(s) = \sum\left(\frac{D}{n}\right)\frac{1}{n^s}$, die bei der Bestimmung der Klassenanzahlen binärer quadratischer Formen auftreten. *Zeits. Math. Phys. 27* (1882), 86-101.

[10] Iwasawa, K. *Lectures on p-Adic L-Functions.* Princeton University Press, Annals Studies *74* (1972).

[11] Jonquière, A. Note sur le série $\sum x^n/n^s$. *Bull. Soc. Math. France 17* (1889), 142-152.

[12] Kubert, D. The universal ordinary distribution. *Bull. Soc. Math. France 107* (1979), 179-202.

[13] Kubert, D. and S. Lang. Units in the modular function field III. Distribution relations. *Math. Ann. 218* (1975), 273-285.

[14] —— Distributions on toroidal groups. *Math. Zeits. 148* (1976), 33-51.

[15] —— *Modular Units.* Grundlehren math. Wiss. 244, Springer, 1981.

[16] LANG, S. Relations de distributions et examples classiques. *Sémin. Delange-Pisot-Poitou 19* (1977-78), n° 40.

[17] —— *Cyclotomic Fields*. Springer, 1978. (See pp. 32, 65.)

[18] —— *Cyclotomic Fields 2*. Springer, 1980. (See Ch. 17.)

[19] LERCH, M. Note sur la fonction $\mathfrak{K}(w, x, s) = \sum_0^\infty \frac{e^{2k\pi i x}}{(w+k)^s}$. *Acta Math. 11* (1887-88), 19-24.

[20] LEWIN, L. *Polylogarithms and Associated Functions*. North-Holland, 1981.

[21] MILNOR, J. Hyperbolic geometry: The first 150 years. *Bull. Amer. Math. Soc. 6* (1982), 9-24.

[22] RAMAKRISHNAN, D. A regulator for curves via the Heisenberg group. *Bull. Amer. Math. Soc. 5* (1981), 191-195. (See also "On the monodromy of higher logarithms", preprint, U. Chicago, 1981.)

[23] SERRE, J.-P. *A Course in Arithmetic*. Springer, 1973.

[24] SHINTANI, T. On a Kronecker limit formula for real quadratic fields. *J. Fac. Sci. Univ. Tokyo, sect. IA Math. 24* (1977), 167-199. (See p. 178.)

[25] SINNOTT, W. On the Stickelberger ideal and the circular units of a cyclotomic field. *Annals of Math. 108* (1978), 107-134.

[26] TRUESDELL, C. On a function which occurs in the theory of the structure of polymers. *Annals of Math. 46* (1945), 144-157.

[27] WEIL, A. *Elliptic functions according to Eisenstein and Kronecker*. Springer, 1976. (See pp. 53-60.)

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