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to make (*) correct. We note that this is equivalent to choosing C_l for $L_n + 1 \leqslant l \leqslant L_{n+1}$. This completes the construction.

To verify that (Q_k) is dense, let E be an entire function, $N > 0$ and $\varepsilon > 0$. Choose n so that

i) $n > N$,

ii) $| h_n(z) - E(z) | \leqslant \frac{1}{3} \varepsilon$ for $| z | \leqslant N$,

iii) $\frac{1}{n} \leqslant \frac{1}{3} \varepsilon$, and

iv) $| (I^{L_n} f)(z) | \leqslant \frac{1}{3} \varepsilon$ for $| z | \leqslant N$.

Then $| Q_{L_n}(z) - E(z) | \leqslant \varepsilon$ for all $| z | \leqslant N$.

We conclude with a result communicated to us by I. N. Baker, namely that there is no entire function f such that the sequence of its compositional iterates $f^{[n]}$ spans a dense set. If f is linear, then the closed span of its iterates contains only linear functions. (That is, the only univalent entire functions are the linear ones. See [BUR], §11.19, p. 370 and the Notes on p. 407 for several proofs.) If f is not linear, then there exist z_1, z_2 with $z_1 \neq z_2$ but $f(z_1) = f(z_2)$. This equality also holds for all linear combinations of iterates, so the closed span of the iterates lies in the proper closed subspace $\{g : g(z_1) = g(z_2)\}$. Finally, we remark that if f is universal in sense (b) then the pair $f(z)$, $z - 1$ *is* universal under composition.

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