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**Artikel:** THE SCHUR SUBGROUP OF THE BRAUER GROUP OF A LOCAL FIELD

### **Bibliographie**

**Autor:** Riehm, C.  
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satisfied. Let the residue class field of  $K(\varepsilon_n)$  have  $2^k$  elements. Set  $n' = (2^k)^{2^h} - 1$ . Then  $n \mid n'$ ,  $n'$  is odd, and  $K(\varepsilon_{n'})/K(\varepsilon_n)$  is unramified of degree  $2^h$ . Consider the conditions (i)-(v) with  $n'$  instead of  $n$ . Then (i) is unchanged, (ii) holds because  $n \mid n'$ , (iii) holds trivially and (v) holds vacuously because  $2^h \mid (K(\varepsilon_{n'}) : K)$ . Finally  $K(\varepsilon_{n'}) \cap K(\varepsilon_4) = K$  since one is ramified and the other is not, so the non-trivial automorphism of  $K(\varepsilon_{4n})/K(\varepsilon_n)$  is the restriction of that of  $K(\varepsilon_{4n'})/K(\varepsilon_{n'})$ , so (iv) holds also for  $n'$ .

We can deduce from this abbreviated form of Janusz' theorem that it is equivalent to Yamada's. Suppose that Janusz' conditions are satisfied, and consider the extension  $\mathbf{Q}_2(\varepsilon_{2^{h+1}}, \varepsilon_n)/K$ . The inertia subgroup of its Galois group is  $\mathcal{g} = \mathcal{G}(\mathbf{Q}_2(\varepsilon_{2^{h+1}}, \varepsilon_n)/K(\varepsilon_n))$ , a group of order 4. Suppose that  $\rho$  is an extension of the non-trivial automorphism of  $\mathbf{Q}_2(\varepsilon_{2^h}, \varepsilon_n)/K(\varepsilon_n)$  to  $\mathbf{Q}_2(\varepsilon_{2^{h+1}}, \varepsilon_n)$ , so  $\rho \in \mathcal{g}$ . By condition (iv), there is an integer  $a \equiv -1 \pmod{2^h}$  such that  $\rho(\varepsilon_{2^{h+1}}) = \varepsilon_{2^{h+1}}^a$ . It follows that  $\rho^2$  is the identity. Thus  $\mathcal{g}$  is non-cyclic. Conversely suppose that there is an extension  $\mathbf{Q}_2(\zeta)/K$  whose inertia subgroup  $\mathcal{g}$  is non-cyclic. As we saw in 1., this means that  $\sigma_{-1}$  is in the Galois group of  $\mathbf{Q}_2^c/K$  and so its restriction (which we also call  $\sigma_{-1}$ ) is in  $\mathcal{G}(\mathbf{Q}_2(\varepsilon_{2^h}, \varepsilon_c)/K)$  and is non-trivial. Its fixed field contains  $K(\varepsilon_c)$ ; by Lemma 3.3 of [J],  $K(\varepsilon_c, \varepsilon_4) = \mathbf{Q}_2(\varepsilon_{2^h}, \varepsilon_c)$  and so the fixed field is *exactly*  $K(\varepsilon_c)$ . Thus both (iv) and (ii) are also fulfilled. (i) holds by Lemma 1.

4. *F. Lorenz*, [L], p. 463. His condition for *non-triviality of  $S(K)$*  is that  $-1$  is a norm in the extension  $K/\mathbf{Q}_2$ . The norm residue symbol in the extension  $\mathbf{Q}_2^c/\mathbf{Q}_2$  sends  $-1$  to  $\sigma_{-1} \in \mathcal{G}(\mathbf{Q}_2^c/\mathbf{Q}_2)$ . Thus it follows from [S], pp. 204-205, that  $-1$  is a norm in  $K/\mathbf{Q}_2$  iff  $\sigma_{-1} \in \mathcal{G}(\mathbf{Q}_2^c/K)$ .

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C. Riehm

Dept. of Mathematics and Statistics  
McMaster University  
Hamilton, Ontario  
Canada L8S 4K1

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