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We complete the proof of the Fundamental Constraint by checking the two degenerate cases, again using Sasaki's equations.

If $p(t)$ is a constant point, then Sasaki's second equation is certainly satisfied, while the first tells us that $(p(t), v(t))$ is a geodesic in US^3 if and only if $v(t)$ traces out, at constant speed, a great circle in the tangent space to S^3 at that point.

If $p(t)$ is a great circle in S^3, travelled at constant speed, then $p'' = 0$, so Sasaki's second equation reads

$$R(v', v)p' = 0 .$$

This can be satisfied in two ways.

One is that $v' = 0$, so that $v(t)$ is a parallel vector field along $p(t)$. In this case, Sasaki's first equation is automatically satisfied, so $(p(t), v(t))$ must be a geodesic in US^3.

The other way for Sasaki's second equation to be satisfied is that v and v' are both orthogonal to p'. Parallel translate $v(t)$ backwards along $p(t)$ to the vector field $u(t)$ in the tangent space to S^3 at $p(0)$. Then Sasaki's first equation says that $u(t)$ traces out, at constant speed, a great circle orthogonal to $p'(0)$. Equivalently, $v(t)$ spins at constant but arbitrary speed along a great circle orthogonal to that of $p(t)$. In these circumstances, the curve $(p(t), v(t))$ will be a geodesic in US^3.

But these are precisely the interpretations of the Fundamental Constraint which were set in the introduction, and the proof is complete.

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