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Artikel: THE EULER CLASS OF ORTHOGONAL RATIONAL REPRESENTATIONS OF FINITE GROUPS

Kapitel: 3. Proof of the main theorem

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$$H := \begin{bmatrix} u_1 C & & & \\ & \cdot & & \\ & & \cdot & \\ & & & \cdot \\ & & & & u_{2n} C \end{bmatrix}$$

is isomorphic to the standard bilinear form if and only if n is even. Let u_{odd} and u_{even} defined by:

$$u_{\text{even}} := \prod_{k=1}^{n} u_{2k} \qquad u_{\text{odd}} := \prod_{k=1}^{n} u_{2k-1};$$

an easy computation shows that $H_v(\xi_{u_{\text{even}}, u_{\text{odd}}}) = H_v(\mu)$ if n is odd. The proposition follows.

3. Proof of the main theorem

(3.1) Lemma. *Let p be a prime number $(p > 2)$. For every integer m satisfying $2m \neq 0 \bmod (p-1) \cdot l_{\mathbf{Q}}(p)$ we have $F_{\mathbf{Q}}(m, p) = 1$.*

Proof. Let G be a p-group, $p > 2$. It follows from (2.2) that any representation ρ of G splits: $\rho = 1 \oplus \tau$ (1 is the 1-dimensional representation of G). Then we have $e(\rho) = e(1)e(\tau) = 0$.

We are now able to prove the main theorem. It has been showed in [3] that $F_{\mathbf{Q}}(n) = 4$ if n is odd. If n is even, four cases have to be distinguished. If $p = 2$ then the $n/2^{N-2}$-fold sum of the irreducible faithful representation of $\mathbf{Z}/2^N$, where 2^N is the 2-primary part of $\operatorname{den}(B_n/n)$, is an orthogonal representation with Euler class of order 2^N (cf. [1]). Let now p be an odd prime. Since the irreducible faithful representation ν of $\mathbf{Z}/p^r (r \geq 1)$ is induced by the irreducible faithful representation of $\mathbf{Z}/p \subset \mathbf{Z}/p^r$, the M-fold sum of ν is equivalent to an orthogonal representation if and only if $l_{\mathbf{Q}}(p)$ divides M. Write $n = Np^k(p-1)$ with g.c.d. $(N, p) = 1$. If N is even, the $2N$-fold sum of the irreducible faithful representation of $\mathbf{Z}/p^{k+1}$ is orthogonal and has Euler class of order p^{k+1} (cf. [1]); if N is odd and $p \neq 7 \bmod 8$ then the $2N$-fold sum of the irreducible faithful representation of $\mathbf{Z}/p^{k+1}$ is orthogonal and has Euler class of order p^{k+1} (cf. [1]). In the three cases, the statement follows from the well known characterization of $\operatorname{den}(B_n/n)$ (cf. [1] for example). Eventually, applying (3.1) we see that $F_{\mathbf{Q}}(n, p) = 1$ if N is odd and $p = 7 \bmod 8$.