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$$H := \begin{bmatrix} u_1 C & & & & \\ & \cdot & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & u_{2n} C \end{bmatrix}$$

is isomorphic to the standard bilinear form if and only if n is even. Let u_{odd} and u_{even} defined by:

$$u_{\text{even}} := \prod_{k=1}^n u_{2k} \quad u_{\text{odd}} := \prod_{k=1}^n u_{2k-1};$$

an easy computation shows that $H_v(\xi_{u_{\text{even}}, u_{\text{odd}}}) = H_v(\mu)$ if n is odd. The proposition follows.

3. PROOF OF THE MAIN THEOREM

(3.1) LEMMA. Let p be a prime number ($p > 2$). For every integer m satisfying $2m \not\equiv 0 \pmod{(p-1) \cdot l_{\mathbf{Q}}(p)}$ we have $F_{\mathbf{Q}}(m, p) = 1$.

Proof. Let G be a p -group, $p > 2$. It follows from (2.2) that any representation ρ of G splits: $\rho = 1 \oplus \tau$ (1 is the 1-dimensional representation of G). Then we have $e(\rho) = e(1)e(\tau) = 0$.

We are now able to prove the main theorem. It has been showed in [3] that $F_{\mathbf{Q}}(n) = 4$ if n is odd. If n is even, four cases have to be distinguished. If $p = 2$ then the $n/2^{N-2}$ -fold sum of the irreducible faithful representation of $\mathbf{Z}/2^N$, where 2^N is the 2-primary part of $\text{den}(B_n/n)$, is an orthogonal representation with Euler class of order 2^N (cf. [1]). Let now p be an odd prime. Since the irreducible faithful representation ν of \mathbf{Z}/p^r ($r \geq 1$) is induced by the irreducible faithful representation of $\mathbf{Z}/p \subset \mathbf{Z}/p^r$, the M -fold sum of ν is equivalent to an orthogonal representation if and only if $l_{\mathbf{Q}}(p)$ divides M . Write $n = Np^k(p-1)$ with $\text{g.c.d.}(N, p) = 1$. If N is even, the $2N$ -fold sum of the irreducible faithful representation of \mathbf{Z}/p^{k+1} is orthogonal and has Euler class of order p^{k+1} (cf. [1]); if N is odd and $p \not\equiv 7 \pmod{8}$ then the $2N$ -fold sum of the irreducible faithful representation of \mathbf{Z}/p^{k+1} is orthogonal and has Euler class of order p^{k+1} (cf. [1]). In the three cases, the statement follows from the well known characterization of $\text{den}(B_n/n)$ (cf. [1] for example). Eventually, applying (3.1) we see that $F_{\mathbf{Q}}(n, p) = 1$ if N is odd and $p \equiv 7 \pmod{8}$.