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AN ELEMENTARY PROOF OF THE STRUCTURE THEOREM FOR CONNECTED SOLVABLE AFFINE ALGEBRAIC GROUPS

by Dragomir Ž. Đoković [1])

Abstract

We give an elementary proof of the basic structure theorem for connected solvable affine algebraic groups G over an algebraically closed field k. The main feature of our proof is that we first establish the important fact that the centralizer in G of a semisimple element s is connected. Then the main structure theorem follows easily. We also prove that such s is contained in a maximal torus and that all maximal tori of G are conjugate. The structure theorem for connected nilpotent affine groups is not needed in the proof; it is obtained at the end as a simple consequence of the main results. In our proof we avoid the use of quotients and Lie algebras of affine groups. On the other hand we use the Lie-Kolchin theorem, Chevalley's theorem, the existence and uniqueness of the Jordan decomposition, and some other elementary facts.

Let k be an algebraically closed field. All algebraic groups will be defined over k and are assumed to be affine. By $N \rtimes H$ we denote the semidirect product of affine algebraic groups where N is a normal and H a complementary subgroup. If G is any affine algebraic group we shall denote by G_u (resp. G_s) the set of all unipotent (resp. semisimple) elements of G. By G^0 we denote the identity component of G and by G' the derived subgroup of G. A torus S in G will be called maximal if $S \subset T$ implies that $T = S$ for any torus T of G. The center of G is denoted by $Z(G)$. The centralizer of $s \in G$ resp. $S \subset G$ in a subgroup $H \subset G$ will be denoted by $Z_H(s)$ resp. $Z_H(S)$. The existence and uniqueness of the Jordan decomposition for elements of G will be used without explicit reference. All group homomorphisms will be homomorphisms of affine algebraic groups. For other proofs of the structure theorem for connected solvable affine algebraic groups we refer the reader to the references [1-5].

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