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since $|\chi| \leqslant 1$ and $|\xi/\theta| \leqslant \sqrt{3}$ for $(\xi/\theta) \in \operatorname{supp} \chi$; this gives the first estimate (4) with $C_{s,t} = 2^{s-t}$.

Similarly, for $s \leqslant t$,

$$(1+|\xi|^2)^s \, | \hat{v}(\xi) - \widehat{S_\theta v}(\xi) |^2 = | 1 - \chi(\xi/\theta) |^2 (1+|\xi|^2)^s \, | \hat{v}(\xi) |^2 ;$$

a Taylor formula gives $| 1 - \chi(\xi/\theta) | \leqslant C_k \, | \xi/\theta |^k$ with $C_k = \sup | \chi^{(k)} | / k!$ for any $k \in \mathbf{N}$ since $\chi(0) = 1$ and $\chi^{(j)}(0) = 0$ for $j > 0$, so that for $t = s + k$

$$\begin{aligned}(1+|\xi|^2)^s \, | \hat{v}(\xi) - \widehat{S_\theta v}(\xi) |^2 &\leqslant C_{t-s}^2 \, | \xi/\theta |^{2(t-s)} (1+|\xi|^2)^s \, | \hat{v}(\xi) |^2 \\ &\leqslant C_{t-s}^2 \theta^{2(s-t)} (1+|\xi|^2)^t \, | \hat{v}(\xi) |^2\end{aligned}$$

whence the second estimate (4) with $C_{s,t} = C_{t-s} = \sup | \chi^{(t-s)} | / (t-s)!$

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