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$$\begin{aligned} \varphi: T_n(q) &\rightarrow \mu_\infty, \\ c^{(1)}(d_{p_1}(\varphi)) &= c^{(2)}(d_{p_2}(\varphi)). \end{aligned}$$

But

$$d_{p_i}(\varphi) = e_{p_i}^{-1} \circ \varphi$$

and

$$c^{(i)}(d_{p_i}(\varphi)) = \varphi^* \circ (e_{p_i}^{-1})^* (e_{p_i}^*(u)) = \varphi^*(u).$$

Remark. It is necessary to reduce to \mathbf{Z}_{l^a} coefficients as the restriction map

$$H^*(Gl_n(q), \hat{\mathbf{Z}}_l) \rightarrow H^*(T_n(q), \hat{\mathbf{Z}}_l)$$

is not injective in general.

SECTION 3. PROOF OF THEOREM 5

CH1 and CH2 clearly follow from resp. CH1 and CH2 in Theorem 2 together with the functoriality of the decomposition map d_p i.e. the diagram

$$\begin{array}{ccc} R_c(G) & \xrightarrow{f^*} & R_c(H) \\ \downarrow d_p & & \downarrow d_p \\ R_p(G) & \xrightarrow{f^*} & R_p(H) \end{array}$$

is commutative for a group homomorphism $f: H \rightarrow G$. To obtain CH3 note that $d_p(\varphi) = e_p^{-1} \circ \varphi$ so by definition

$$c_1(\varphi) = c_1(d_p(\varphi)) = (e_p^{-1} \circ \varphi)^*(e_p^*(u)) = \varphi^* \circ (e_p^{-1})^* \circ e_p^*(u) = \varphi^*(u).$$

Furthermore let δ be the connecting homomorphism obtained from the exact sequence

$$\mathbf{Z} \rightarrow \mathbf{Q} \rightarrow \mathbf{Q}/\mathbf{Z}$$

As the diagram

$$\begin{array}{ccc} H^1(\mu_\infty, \mathbf{Q}/\mathbf{Z}) & \xrightarrow{\sim \delta} & H^2(\mu_\infty, \mathbf{Z}) \\ \downarrow \varphi^* & & \downarrow \varphi^* \\ H^1(G, \mathbf{Q}/\mathbf{Z}) & \xrightarrow{\sim \delta} & H^2(G, \mathbf{Z}) \end{array}$$

is commutative and as both δ 's are isomorphisms it suffices to show that

$$\delta^{-1} \circ c_1: \text{Hom}(G, \mathbf{C}^*) \rightarrow H^1(G, \mathbf{Q}/\mathbf{Z}) \simeq \text{Hom}(G, \mathbf{Q}/\mathbf{Z})$$

is an isomorphism.

But by inspection $\delta^{-1} \circ c_1(\varphi) = \delta^{-1}(u) \circ \varphi$, and as u is a $\hat{\mathbf{Z}} = \text{End}_{\mathbf{Z}}(\mu_{\infty})$ generator for $H^2(\mu_{\infty}, \mathbf{Z}) \simeq \hat{\mathbf{Z}}$, $\delta^{-1}(u)$ is an isomorphism

$$\delta^{-1}(u): \mu_{\infty} \xrightarrow{\sim} \mathbf{Q}/\mathbf{Z}.$$

SECTION 4. CHERN CLASSES FOR LOCALLY FINITE GROUPS

The definition will be based on the following two observations. In the following, let $G = \varinjlim G_k$ be a locally finite group where $\{G_k\}$ is a family of finite subgroups.

LEMMA. *Let*

$$\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$$

be a representation of G . Then φ is uniquely determined by its restrictions

$$\varphi_k: G_k \rightarrow \text{Gl}_n(\mathbf{C}).$$

Conversely given a family of compatible representations $\varphi_k: G_k \rightarrow \text{Gl}_n(\mathbf{C})$, there exists a unique $\varphi: G \rightarrow \text{Gl}_n(\mathbf{C})$ which restricts to φ_k for all k .

Proof. From the universal property of the direct limit, we have

$$\text{Hom}(G, \text{Gl}_n(\mathbf{C})) \cong \varprojlim \text{Hom}(G_k, \text{Gl}_n(\mathbf{C})).$$

PROPOSITION. *For all $i \geq 0$, the natural map*

$$H^i(G, \mathbf{Z}) \cong \varprojlim H^i(G_k, \mathbf{Z}).$$

is an isomorphism.

Proof. Obvious for $i = 0, 1$. For $i \geq 1$, the homology groups

$$H_i(G, \mathbf{Z}) = \varinjlim H_i(G_k, \mathbf{Z})$$

are all abelian torsion groups.

Now by the universal coefficient theorem ($i \geq 1$)