Zeitschrift: L'Enseignement Mathématique

Band: 35 (1989)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON SUMS OF FOURIER COEFFICIENTS OF CUSP FORMS

Kurzfassung

Autor: Hafner, James Lee / Ivi, Aleksandar

DOI: https://doi.org/10.5169/seals-57381

L'Enseignement Mathématique, t. 35 (1989), p. 375-382

ON SUMS OF FOURIER COEFFICIENTS OF CUSP FORMS

by James Lee HAFNER and Aleksandar IVIĆ

ABSTRACT. In this paper, we provide both upper and lower estimates for the summatory functions of coefficients of cusp forms. Some of the results also hold for Maass wave forms. The proofs are essentially applications of general results of Chandrasekharan and Narasimhan and of Redmond together with some non-trivial results of Murty, Rankin and others.

The purpose of this note is to collect some hitherto unnoticed or unpublished results concerning the summatory functions of the Fourier coefficients of cusp forms, including Maass wave forms. Most of the results are obtained by direct application of some general theorems about Dirichlet series satisfying a functional equation and more specific (and deeper) results concerning cusp form coefficients. One of our purposes in writing this paper is to get some of the "folklore" into print.

As usual, we will need some notation. Let $F(z)$ be a cusp form of weight k for $\Gamma = PSL(2, Z)$. (Our results also hold for cusp forms on congruence subgroups but this extra generality would only make the notation more complicated.) Write $F(z)$ in a Fourier series:

$$F(z) = \sum_{n=1}^{\infty} a(n) e^{2\pi i n z} . \qquad (\text{Im } z > 0)$$

If we assume that F is a normalized eigenfunction for the Hecke operators, then we have that the $a(n)$'s are multiplicative, real valued, and satisfy

$$| a(n) | \leqslant d(n) n^{(k-1)/2} ,$$

and

$$(1) \qquad \sum_{n \leqslant x} a(n)^2 = A x^k + O(x^{k-2/5}) .$$