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the Fourier transform $\hat{1}_E$ does not vanish on any nonempty open subset of the dual group $\hat{G}$. Thus bounded Borel subsets of $\mathbf{R}^n$ of positive Lebesgue measure are determining sets by the analyticity of $\hat{1}_E$. Classical quasi-analyticity results apply to give conditions on the growth of an unbounded subset $E \subseteq \mathbf{R}^n$ to be a determining set. Settling a problem that was open for some time, Kargaev ([17]) proved the existence of sets $E \subseteq \mathbf{R}^n$ of finite Lebesgue measure which are not determining sets for finite complex measures.

The problem of determining sets has also been studied with the class of probability measures replaced by other classes of measures, e.g., a class of infinite measures with growth/decay conditions (see [22], [11] and [28]). Also different groups of homeomorphisms acting on X have been considered in these studies.

Finally, we refer to the following form of the *support problem* analogous to the well known problem in the case of Radon transform. Let X be a symmetric space (Euclidean, compact or non-compact). Let x_0 be a fixed point of X. If E is a relatively compact subset of positive measure and if $\int_{gE} f = 0$ for all $g \in G$ with $d(x_0, gx_0) > R$ what can one say about the support of f with respect to the reference point x_0? (Here, d stands for the geodesic distance.) Some partial answers to this question are known (see [26] and [28]).

We have not addressed ourselves in this paper to the situation when X is an infinite-dimensional Hilbert space or X is an arbitrary Riemannian manifold. Another important problem we have not considered is the *local version* of the Pompeiu problem. (For this, we refer the reader to [5] and [6]). We have restricted ourselves to the situation of symmetric spaces and locally compact groups and the relationship of the Pompeiu problem with harmonic analysis and representation theory.

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